

# **MonoidalCategories**

**Monoidal and monoidal (co)closed  
categories**

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# Chapter 1

## Monoidal Categories

### 1.1 Monoidal Categories

A 6-tuple  $(\mathbf{C}, \otimes, 1, \alpha, \lambda, \rho)$  consisting of

- a category  $\mathbf{C}$ ,
- a functor  $\otimes : \mathbf{C} \times \mathbf{C} \rightarrow \mathbf{C}$  compatible with the congruence of morphisms,
- an object  $1 \in \mathbf{C}$ ,
- a natural isomorphism  $\alpha_{a,b,c} : a \otimes (b \otimes c) \cong (a \otimes b) \otimes c$ ,
- a natural isomorphism  $\lambda_a : 1 \otimes a \cong a$ ,
- a natural isomorphism  $\rho_a : a \otimes 1 \cong a$ ,

is called a *monoidal category*, if

- for all objects  $a, b, c, d$ , the pentagon identity holds:

$$(\alpha_{a,b,c} \otimes \text{id}_d) \circ \alpha_{a,b \otimes c,d} \circ (\text{id}_a \otimes \alpha_{b,c,d}) \sim \alpha_{a \otimes b,c,d} \circ \alpha_{a,b,c \otimes d},$$

- for all objects  $a, c$ , the triangle identity holds:

$$(\rho_a \otimes \text{id}_c) \circ \alpha_{a,1,c} \sim \text{id}_a \otimes \lambda_c.$$

The corresponding GAP property is given by `IsMonoidalCategory`.

#### 1.1.1 TensorProductOnMorphisms (for IsCapCategoryMorphism, IsCapCategory-Morphism)

▷ `TensorProductOnMorphisms(alpha, beta)` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, a' \otimes b')$

The arguments are two morphisms  $\alpha : a \rightarrow a', \beta : b \rightarrow b'$ . The output is the tensor product  $\alpha \otimes \beta$ .

### 1.1.2 TensorProductOnMorphismsWithGivenTensorProducts (for IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryMorphism, IsCapCategoryObject)

▷ TensorProductOnMorphismsWithGivenTensorProducts( $s$ ,  $\alpha$ ,  $\beta$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, a' \otimes b')$

The arguments are an object  $s = a \otimes b$ , two morphisms  $\alpha : a \rightarrow a'$ ,  $\beta : b \rightarrow b'$ , and an object  $r = a' \otimes b'$ . The output is the tensor product  $\alpha \otimes \beta$ .

### 1.1.3 AssociatorRightToLeft (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ AssociatorRightToLeft( $a$ ,  $b$ ,  $c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes (b \otimes c), (a \otimes b) \otimes c)$ .

The arguments are three objects  $a, b, c$ . The output is the associator  $\alpha_{a,(b,c)} : a \otimes (b \otimes c) \rightarrow (a \otimes b) \otimes c$ .

### 1.1.4 AssociatorRightToLeftWithGivenTensorProducts (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ AssociatorRightToLeftWithGivenTensorProducts( $s$ ,  $a$ ,  $b$ ,  $c$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes (b \otimes c), (a \otimes b) \otimes c)$ .

The arguments are an object  $s = a \otimes (b \otimes c)$ , three objects  $a, b, c$ , and an object  $r = (a \otimes b) \otimes c$ . The output is the associator  $\alpha_{a,(b,c)} : a \otimes (b \otimes c) \rightarrow (a \otimes b) \otimes c$ .

### 1.1.5 AssociatorLeftToRight (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ AssociatorLeftToRight( $a$ ,  $b$ ,  $c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}((a \otimes b) \otimes c \rightarrow a \otimes (b \otimes c))$ .

The arguments are three objects  $a, b, c$ . The output is the associator  $\alpha_{(a,b),c} : (a \otimes b) \otimes c \rightarrow a \otimes (b \otimes c)$ .

### 1.1.6 AssociatorLeftToRightWithGivenTensorProducts (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ AssociatorLeftToRightWithGivenTensorProducts( $s$ ,  $a$ ,  $b$ ,  $c$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}((a \otimes b) \otimes c \rightarrow a \otimes (b \otimes c))$ .

The arguments are an object  $s = (a \otimes b) \otimes c$ , three objects  $a, b, c$ , and an object  $r = a \otimes (b \otimes c)$ . The output is the associator  $\alpha_{(a,b),c} : (a \otimes b) \otimes c \rightarrow a \otimes (b \otimes c)$ .

### 1.1.7 LeftUnitor (for IsCapCategoryObject)

▷ LeftUnitor( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(1 \otimes a, a)$

The argument is an object  $a$ . The output is the left unitor  $\lambda_a : 1 \otimes a \rightarrow a$ .

### 1.1.8 LeftUnitorWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject)

▷ LeftUnitorWithGivenTensorProduct( $a, s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(1 \otimes a, a)$

The arguments are an object  $a$  and an object  $s = 1 \otimes a$ . The output is the left unitor  $\lambda_a : 1 \otimes a \rightarrow a$ .

### 1.1.9 LeftUnitorInverse (for IsCapCategoryObject)

▷ LeftUnitorInverse( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(a, 1 \otimes a)$

The argument is an object  $a$ . The output is the inverse of the left unitor  $\lambda_a^{-1} : a \rightarrow 1 \otimes a$ .

### 1.1.10 LeftUnitorInverseWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject)

▷ LeftUnitorInverseWithGivenTensorProduct( $a, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, 1 \otimes a)$

The argument is an object  $a$  and an object  $r = 1 \otimes a$ . The output is the inverse of the left unitor  $\lambda_a^{-1} : a \rightarrow 1 \otimes a$ .

### 1.1.11 RightUnitor (for IsCapCategoryObject)

▷ RightUnitor( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(a \otimes 1, a)$

The argument is an object  $a$ . The output is the right unitor  $\rho_a : a \otimes 1 \rightarrow a$ .

### 1.1.12 RightUnitorWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject)

▷ RightUnitorWithGivenTensorProduct( $a, s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes 1, a)$

The arguments are an object  $a$  and an object  $s = a \otimes 1$ . The output is the right unitor  $\rho_a : a \otimes 1 \rightarrow a$ .

### 1.1.13 RightUnitorInverse (for IsCapCategoryObject)

▷ RightUnitorInverse( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(a, a \otimes 1)$

The argument is an object  $a$ . The output is the inverse of the right unitor  $\rho_a^{-1} : a \rightarrow a \otimes 1$ .

### 1.1.14 RightUnitorInverseWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject)

▷ RightUnitorInverseWithGivenTensorProduct( $a, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, a \otimes 1)$

The arguments are an object  $a$  and an object  $r = a \otimes 1$ . The output is the inverse of the right unitor  $\rho_a^{-1} : a \rightarrow a \otimes 1$ .

### 1.1.15 TensorProductOnObjects (for IsCapCategoryObject, IsCapCategoryObject)

▷ `TensorProductOnObjects(a, b)` (operation)

**Returns:** an object

The arguments are two objects  $a, b$ . The output is the tensor product  $a \otimes b$ .

### 1.1.16 AddTensorProductOnObjects (for IsCapCategory, IsFunction)

▷ `AddTensorProductOnObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductOnObjects`.  $F : (a, b) \mapsto a \otimes b$ .

### 1.1.17 TensorUnit (for IsCapCategory)

▷ `TensorUnit(C)` (attribute)

**Returns:** an object

The argument is a category  $C$ . The output is the tensor unit  $1$  of  $C$ .

### 1.1.18 AddTensorUnit (for IsCapCategory, IsFunction)

▷ `AddTensorUnit(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorUnit`.  $F : () \mapsto 1$ .

## 1.2 Additive Monoidal Categories

### 1.2.1 LeftDistributivityExpanding (for IsCapCategoryObject, IsList)

▷ `LeftDistributivityExpanding(a, L)` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes (b_1 \oplus \dots \oplus b_n), (a \otimes b_1) \oplus \dots \oplus (a \otimes b_n))$

The arguments are an object  $a$  and a list of objects  $L = (b_1, \dots, b_n)$ . The output is the left distributivity morphism  $a \otimes (b_1 \oplus \dots \oplus b_n) \rightarrow (a \otimes b_1) \oplus \dots \oplus (a \otimes b_n)$ .

### 1.2.2 LeftDistributivityExpandingWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ `LeftDistributivityExpandingWithGivenObjects(s, a, L, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$

The arguments are an object  $s = a \otimes (b_1 \oplus \dots \oplus b_n)$ , an object  $a$ , a list of objects  $L = (b_1, \dots, b_n)$ , and an object  $r = (a \otimes b_1) \oplus \dots \oplus (a \otimes b_n)$ . The output is the left distributivity morphism  $s \rightarrow r$ .

### 1.2.3 LeftDistributivityFactoring (for IsCapCategoryObject, IsList)

▷ `LeftDistributivityFactoring(a, L)` (operation)

**Returns:** a morphism in  $\text{Hom}((a \otimes b_1) \oplus \dots \oplus (a \otimes b_n), a \otimes (b_1 \oplus \dots \oplus b_n))$

The arguments are an object  $a$  and a list of objects  $L = (b_1, \dots, b_n)$ . The output is the left distributivity morphism  $(a \otimes b_1) \oplus \dots \oplus (a \otimes b_n) \rightarrow a \otimes (b_1 \oplus \dots \oplus b_n)$ .

#### 1.2.4 LeftDistributivityFactoringWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ LeftDistributivityFactoringWithGivenObjects( $s$ ,  $a$ ,  $L$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$

The arguments are an object  $s = (a \otimes b_1) \oplus \dots \oplus (a \otimes b_n)$ , an object  $a$ , a list of objects  $L = (b_1, \dots, b_n)$ , and an object  $r = a \otimes (b_1 \oplus \dots \oplus b_n)$ . The output is the left distributivity morphism  $s \rightarrow r$ .

#### 1.2.5 RightDistributivityExpanding (for IsList, IsCapCategoryObject)

▷ RightDistributivityExpanding( $L$ ,  $a$ ) (operation)

**Returns:** a morphism in  $\text{Hom}((b_1 \oplus \dots \oplus b_n) \otimes a, (b_1 \otimes a) \oplus \dots \oplus (b_n \otimes a))$

The arguments are a list of objects  $L = (b_1, \dots, b_n)$  and an object  $a$ . The output is the right distributivity morphism  $(b_1 \oplus \dots \oplus b_n) \otimes a \rightarrow (b_1 \otimes a) \oplus \dots \oplus (b_n \otimes a)$ .

#### 1.2.6 RightDistributivityExpandingWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject, IsCapCategoryObject)

▷ RightDistributivityExpandingWithGivenObjects( $s$ ,  $L$ ,  $a$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$

The arguments are an object  $s = (b_1 \oplus \dots \oplus b_n) \otimes a$ , a list of objects  $L = (b_1, \dots, b_n)$ , an object  $a$ , and an object  $r = (b_1 \otimes a) \oplus \dots \oplus (b_n \otimes a)$ . The output is the right distributivity morphism  $s \rightarrow r$ .

#### 1.2.7 RightDistributivityFactoring (for IsList, IsCapCategoryObject)

▷ RightDistributivityFactoring( $L$ ,  $a$ ) (operation)

**Returns:** a morphism in  $\text{Hom}((b_1 \otimes a) \oplus \dots \oplus (b_n \otimes a), (b_1 \oplus \dots \oplus b_n) \otimes a)$

The arguments are a list of objects  $L = (b_1, \dots, b_n)$  and an object  $a$ . The output is the right distributivity morphism  $(b_1 \otimes a) \oplus \dots \oplus (b_n \otimes a) \rightarrow (b_1 \oplus \dots \oplus b_n) \otimes a$ .

#### 1.2.8 RightDistributivityFactoringWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject, IsCapCategoryObject)

▷ RightDistributivityFactoringWithGivenObjects( $s$ ,  $L$ ,  $a$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$

The arguments are an object  $s = (b_1 \otimes a) \oplus \dots \oplus (b_n \otimes a)$ , a list of objects  $L = (b_1, \dots, b_n)$ , an object  $a$ , and an object  $r = (b_1 \oplus \dots \oplus b_n) \otimes a$ . The output is the right distributivity morphism  $s \rightarrow r$ .

### 1.3 Braided Monoidal Categories

A monoidal category  $\mathbf{C}$  equipped with a natural isomorphism  $B_{a,b} : a \otimes b \cong b \otimes a$  is called a *braided monoidal category* if

$$\bullet \lambda_a \circ B_{a,1} \sim \rho_a,$$



- $(B_{c,a} \otimes \text{id}_b) \circ \alpha_{c,a,b} \circ B_{a \otimes b, c} \sim \alpha_{a,c,b} \circ (\text{id}_a \otimes B_{b,c}) \circ \alpha_{a,b,c}^{-1}$ ,
- $(\text{id}_b \otimes B_{c,a}) \circ \alpha_{b,c,a}^{-1} \circ B_{a,b \otimes c} \sim \alpha_{b,a,c}^{-1} \circ (B_{a,b} \otimes \text{id}_c) \circ \alpha_{a,b,c}$ .

The corresponding GAP property is given by `IsBraidedMonoidalCategory`.

### 1.3.1 Braiding (for `IsCapCategoryObject`, `IsCapCategoryObject`)

▷ `Braiding(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, b \otimes a)$ .

The arguments are two objects  $a, b$ . The output is the braiding  $B_{a,b} : a \otimes b \rightarrow b \otimes a$ .

### 1.3.2 BraidingWithGivenTensorProducts (for `IsCapCategoryObject`, `IsCapCategoryObject`, `IsCapCategoryObject`)

▷ `BraidingWithGivenTensorProducts(s, a, b, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, b \otimes a)$ .

The arguments are an object  $s = a \otimes b$ , two objects  $a, b$ , and an object  $r = b \otimes a$ . The output is the braiding  $B_{a,b} : a \otimes b \rightarrow b \otimes a$ .

### 1.3.3 BraidingInverse (for `IsCapCategoryObject`, `IsCapCategoryObject`)

▷ `BraidingInverse(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(b \otimes a, a \otimes b)$ .

The arguments are two objects  $a, b$ . The output is the inverse braiding  $B_{a,b}^{-1} : b \otimes a \rightarrow a \otimes b$ .

### 1.3.4 BraidingInverseWithGivenTensorProducts (for `IsCapCategoryObject`, `IsCapCategoryObject`, `IsCapCategoryObject`)

▷ `BraidingInverseWithGivenTensorProducts(s, a, b, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(b \otimes a, a \otimes b)$ .

The arguments are an object  $s = b \otimes a$ , two objects  $a, b$ , and an object  $r = a \otimes b$ . The output is the inverse braiding  $B_{a,b}^{-1} : b \otimes a \rightarrow a \otimes b$ .

## 1.4 Symmetric Monoidal Categories

A braided monoidal category  $\mathbf{C}$  is called *symmetric monoidal category* if  $B_{a,b}^{-1} \sim B_{b,a}$ . The corresponding GAP property is given by `IsSymmetricMonoidalCategory`.

## 1.5 Left Closed Monoidal Categories

A monoidal category  $\mathbf{C}$  which has for each functor  $- \otimes b : \mathbf{C} \rightarrow \mathbf{C}$  a right adjoint (denoted by  $\underline{\text{Hom}}_\ell(b, -)$ ) is called a *left closed monoidal category*.

If no operations involving left duals are installed manually, the left dual objects will be derived as  $a^\vee := \underline{\text{Hom}}_\ell(a, 1)$ .

The corresponding GAP property is called `IsLeftClosedMonoidalCategory`.

### 1.5.1 LeftInternalHomOnObjects (for IsCapCategoryObject, IsCapCategoryObject)

▷ LeftInternalHomOnObjects( $a, b$ ) (operation)

**Returns:** an object

The arguments are two objects  $a, b$ . The output is the internal hom object  $\underline{\text{Hom}}_\ell(a, b)$ .

### 1.5.2 LeftInternalHomOnMorphisms (for IsCapCategoryMorphism, IsCapCategoryMorphism)

▷ LeftInternalHomOnMorphisms( $\alpha, \beta$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}_\ell(a', b), \underline{\text{Hom}}_\ell(a, b'))$

The arguments are two morphisms  $\alpha : a \rightarrow a', \beta : b \rightarrow b'$ . The output is the internal hom morphism  $\underline{\text{Hom}}_\ell(\alpha, \beta) : \underline{\text{Hom}}_\ell(a', b) \rightarrow \underline{\text{Hom}}_\ell(a, b')$ .

### 1.5.3 LeftInternalHomOnMorphismsWithGivenLeftInternalHoms (for IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryMorphism, IsCapCategoryObject)

▷ LeftInternalHomOnMorphismsWithGivenLeftInternalHoms( $s, \alpha, \beta, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$

The arguments are an object  $s = \underline{\text{Hom}}_\ell(a', b)$ , two morphisms  $\alpha : a \rightarrow a', \beta : b \rightarrow b'$ , and an object  $r = \underline{\text{Hom}}_\ell(a, b')$ . The output is the internal hom morphism  $\underline{\text{Hom}}_\ell(\alpha, \beta) : \underline{\text{Hom}}_\ell(a', b) \rightarrow \underline{\text{Hom}}_\ell(a, b')$ .

### 1.5.4 LeftClosedMonoidalEvaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ LeftClosedMonoidalEvaluationMorphism( $a, b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}_\ell(a, b) \otimes a, b)$ .

The arguments are two objects  $a, b$ . The output is the evaluation morphism  $\text{ev}_{a,b} : \underline{\text{Hom}}_\ell(a, b) \otimes a \rightarrow b$ , i.e., the counit of the tensor hom adjunction.

### 1.5.5 LeftClosedMonoidalEvaluationMorphismWithGivenSource (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftClosedMonoidalEvaluationMorphismWithGivenSource( $a, b, s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, b)$ .

The arguments are two objects  $a, b$  and an object  $s = \underline{\text{Hom}}_\ell(a, b) \otimes a$ . The output is the evaluation morphism  $\text{ev}_{a,b} : \underline{\text{Hom}}_\ell(a, b) \otimes a \rightarrow b$ , i.e., the counit of the tensor hom adjunction.

### 1.5.6 LeftClosedMonoidalCoevaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ LeftClosedMonoidalCoevaluationMorphism( $a, b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, \underline{\text{Hom}}_\ell(a, b \otimes a))$ .

The arguments are two objects  $a, b$ . The output is the coevaluation morphism  $\text{coev}_{a,b} : b \rightarrow \underline{\text{Hom}}_\ell(a, b \otimes a)$ , i.e., the unit of the tensor hom adjunction.

### 1.5.7 LeftClosedMonoidalCoevaluationMorphismWithGivenRange (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftClosedMonoidalCoevaluationMorphismWithGivenRange( $a, b, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, r)$ .

The arguments are two objects  $a, b$  and an object  $r = \underline{\text{Hom}}_\ell(a, b \otimes a)$ . The output is the coevaluation morphism  $\text{coev}_{a,b} : b \rightarrow \underline{\text{Hom}}_\ell(a, b \otimes a)$ , i.e., the unit of the tensor hom adjunction.

### 1.5.8 TensorProductToLeftInternalHomAdjunctMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ TensorProductToLeftInternalHomAdjunctMorphism( $a, b, f$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{Hom}}_\ell(b, c))$ .

The arguments are two objects  $a, b$  and a morphism  $f : a \otimes b \rightarrow c$ . The output is a morphism  $g : a \rightarrow \underline{\text{Hom}}_\ell(b, c)$  corresponding to  $f$  under the tensor hom adjunction.

### 1.5.9 TensorProductToLeftInternalHomAdjunctMorphismWithGivenLeftInternalHom (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ TensorProductToLeftInternalHomAdjunctMorphismWithGivenLeftInternalHom( $a, b, f, i$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, i)$ .

The arguments are two objects  $a, b$ , a morphism  $f : a \otimes b \rightarrow c$  and an object  $i = \underline{\text{Hom}}_\ell(b, c)$ . The output is a morphism  $g : a \rightarrow \underline{\text{Hom}}_\ell(b, c)$  corresponding to  $f$  under the tensor hom adjunction.

### 1.5.10 LeftInternalHomToTensorProductAdjunctMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ LeftInternalHomToTensorProductAdjunctMorphism( $b, c, g$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, c)$ .

The arguments are two objects  $b, c$  and a morphism  $g : a \rightarrow \underline{\text{Hom}}_\ell(b, c)$ . The output is a morphism  $f : a \otimes b \rightarrow c$  corresponding to  $g$  under the tensor hom adjunction.

### 1.5.11 LeftInternalHomToTensorProductAdjunctMorphismWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ LeftInternalHomToTensorProductAdjunctMorphismWithGivenTensorProduct( $b, c, g, t$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(t, c)$ .

The arguments are two objects  $b, c$ , a morphism  $g : a \rightarrow \underline{\text{Hom}}_\ell(b, c)$  and an object  $t = a \otimes b$ . The output is a morphism  $f : a \otimes b \rightarrow c$  corresponding to  $g$  under the tensor hom adjunction.

### 1.5.12 LeftClosedMonoidalPreComposeMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftClosedMonoidalPreComposeMorphism( $a, b, c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}_\ell(a, b) \otimes \underline{\text{Hom}}_\ell(b, c), \underline{\text{Hom}}_\ell(a, c))$ .

The arguments are three objects  $a, b, c$ . The output is the precomposition morphism  $\text{LeftClosedMonoidalPreComposeMorphism}_{a,b,c} : \underline{\text{Hom}}_\ell(a, b) \otimes \underline{\text{Hom}}_\ell(b, c) \rightarrow \underline{\text{Hom}}_\ell(a, c)$ .

### 1.5.13 LeftClosedMonoidalPreComposeMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftClosedMonoidalPreComposeMorphismWithGivenObjects( $s, a, b, c, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = \underline{\text{Hom}}_\ell(a, b) \otimes \underline{\text{Hom}}_\ell(b, c)$ , three objects  $a, b, c$ , and an object  $r = \underline{\text{Hom}}_\ell(a, c)$ . The output is the precomposition morphism  $\text{LeftClosedMonoidalPreComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{Hom}}_\ell(a, b) \otimes \underline{\text{Hom}}_\ell(b, c) \rightarrow \underline{\text{Hom}}_\ell(a, c)$ .

### 1.5.14 LeftClosedMonoidalPostComposeMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftClosedMonoidalPostComposeMorphism( $a, b, c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}_\ell(b, c) \otimes \underline{\text{Hom}}_\ell(a, b), \underline{\text{Hom}}_\ell(a, c))$ .

The arguments are three objects  $a, b, c$ . The output is the postcomposition morphism  $\text{LeftClosedMonoidalPostComposeMorphism}_{a,b,c} : \underline{\text{Hom}}_\ell(b, c) \otimes \underline{\text{Hom}}_\ell(a, b) \rightarrow \underline{\text{Hom}}_\ell(a, c)$ .

### 1.5.15 LeftClosedMonoidalPostComposeMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftClosedMonoidalPostComposeMorphismWithGivenObjects( $s, a, b, c, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = \underline{\text{Hom}}_\ell(b, c) \otimes \underline{\text{Hom}}_\ell(a, b)$ , three objects  $a, b, c$ , and an object  $r = \underline{\text{Hom}}_\ell(a, c)$ . The output is the postcomposition morphism  $\text{LeftClosedMonoidalPostComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{Hom}}_\ell(b, c) \otimes \underline{\text{Hom}}_\ell(a, b) \rightarrow \underline{\text{Hom}}_\ell(a, c)$ .

### 1.5.16 LeftDualOnObjects (for IsCapCategoryObject)

▷ LeftDualOnObjects( $a$ ) (attribute)

**Returns:** an object

The argument is an object  $a$ . The output is its dual object  $a^\vee$ .

### 1.5.17 LeftDualOnMorphisms (for IsCapCategoryMorphism)

▷ LeftDualOnMorphisms( $\alpha$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(b^\vee, a^\vee)$ .

The argument is a morphism  $\alpha : a \rightarrow b$ . The output is its dual morphism  $\alpha^\vee : b^\vee \rightarrow a^\vee$ .

### 1.5.18 LeftDualOnMorphismsWithGivenLeftDuals (for IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ LeftDualOnMorphismsWithGivenLeftDuals( $s$ ,  $\alpha$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The argument is an object  $s = b^\vee$ , a morphism  $\alpha : a \rightarrow b$ , and an object  $r = a^\vee$ . The output is the dual morphism  $\alpha^\vee : b^\vee \rightarrow a^\vee$ .

### 1.5.19 LeftClosedMonoidalEvaluationForLeftDual (for IsCapCategoryObject)

▷ LeftClosedMonoidalEvaluationForLeftDual( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes a, 1)$ .

The argument is an object  $a$ . The output is the evaluation morphism  $\text{ev}_a : a^\vee \otimes a \rightarrow 1$ .

### 1.5.20 LeftClosedMonoidalEvaluationForLeftDualWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftClosedMonoidalEvaluationForLeftDualWithGivenTensorProduct( $s$ ,  $a$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = a^\vee \otimes a$ , an object  $a$ , and an object  $r = 1$ . The output is the evaluation morphism  $\text{ev}_a : a^\vee \otimes a \rightarrow 1$ .

### 1.5.21 MorphismToLeftBidual (for IsCapCategoryObject)

▷ MorphismToLeftBidual( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(a, (a^\vee)^\vee)$ .

The argument is an object  $a$ . The output is the morphism to the bidual  $a \rightarrow (a^\vee)^\vee$ .

### 1.5.22 MorphismToLeftBidualWithGivenLeftBidual (for IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismToLeftBidualWithGivenLeftBidual( $a$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, r)$ .

The arguments are an object  $a$ , and an object  $r = (a^\vee)^\vee$ . The output is the morphism to the bidual  $a \rightarrow (a^\vee)^\vee$ .

### 1.5.23 TensorProductLeftInternalHomCompatibilityMorphism (for IsList)

▷ TensorProductLeftInternalHomCompatibilityMorphism( $list$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}_\ell(a, a') \otimes \underline{\text{Hom}}_\ell(b, b'), \underline{\text{Hom}}_\ell(a \otimes b, a' \otimes b'))$ .

The argument is a list of four objects  $[a, a', b, b']$ . The output is the natural morphism  $\text{TensorProductLeftInternalHomCompatibilityMorphism}_{a, a', b, b'} : \underline{\text{Hom}}_\ell(a, a') \otimes \underline{\text{Hom}}_\ell(b, b') \rightarrow \underline{\text{Hom}}_\ell(a \otimes b, a' \otimes b')$ .

### 1.5.24 TensorProductLeftInternalHomCompatibilityMorphismWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ TensorProductLeftInternalHomCompatibilityMorphismWithGivenObjects( $s$ ,  $list$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are a list of four objects  $[a, a', b, b']$ , and two objects  $s = \underline{\text{Hom}}_\ell(a, a') \otimes \underline{\text{Hom}}_\ell(b, b')$  and  $r = \underline{\text{Hom}}_\ell(a \otimes b, a' \otimes b')$ . The output is the natural morphism  $\text{TensorProductLeftInternalHomCompatibilityMorphismWithGivenObjects}_{a, a', b, b'} : \underline{\text{Hom}}_\ell(a, a') \otimes \underline{\text{Hom}}_\ell(b, b') \rightarrow \underline{\text{Hom}}_\ell(a \otimes b, a' \otimes b')$ .

### 1.5.25 TensorProductLeftDualityCompatibilityMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ TensorProductLeftDualityCompatibilityMorphism( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes b^\vee, (a \otimes b)^\vee)$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{TensorProductLeftDualityCompatibilityMorphism} : a^\vee \otimes b^\vee \rightarrow (a \otimes b)^\vee$ .

### 1.5.26 TensorProductLeftDualityCompatibilityMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ TensorProductLeftDualityCompatibilityMorphismWithGivenObjects( $s$ ,  $a$ ,  $b$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = a^\vee \otimes b^\vee$ , two objects  $a, b$ , and an object  $r = (a \otimes b)^\vee$ . The output is the natural morphism  $\text{TensorProductLeftDualityCompatibilityMorphismWithGivenObjects}_{a, b} : a^\vee \otimes b^\vee \rightarrow (a \otimes b)^\vee$ .

### 1.5.27 MorphismFromTensorProductToLeftInternalHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromTensorProductToLeftInternalHom( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes b, \underline{\text{Hom}}_\ell(a, b))$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{MorphismFromTensorProductToLeftInternalHom}_{a, b} : a^\vee \otimes b \rightarrow \underline{\text{Hom}}_\ell(a, b)$ .

### 1.5.28 MorphismFromTensorProductToLeftInternalHomWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromTensorProductToLeftInternalHomWithGivenObjects( $s$ ,  $a$ ,  $b$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = a^\vee \otimes b$ , two objects  $a, b$ , and an object  $r = \underline{\text{Hom}}_\ell(a, b)$ . The output is the natural morphism  $\text{MorphismFromTensorProductToLeftInternalHomWithGivenObjects}_{a, b} : a^\vee \otimes b \rightarrow \underline{\text{Hom}}_\ell(a, b)$ .

### 1.5.29 IsomorphismFromLeftDualObjectToLeftInternalHomIntoTensorUnit (for IsCapCategoryObject)

▷ `IsomorphismFromLeftDualObjectToLeftInternalHomIntoTensorUnit(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a^\vee, \underline{\text{Hom}}_\ell(a, 1))$ .

The argument is an object  $a$ . The output is the isomorphism  $\text{IsomorphismFromLeftDualObjectToLeftInternalHomIntoTensorUnit}_a : a^\vee \rightarrow \underline{\text{Hom}}_\ell(a, 1)$ .

### 1.5.30 IsomorphismFromLeftInternalHomIntoTensorUnitToLeftDualObject (for IsCapCategoryObject)

▷ `IsomorphismFromLeftInternalHomIntoTensorUnitToLeftDualObject(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}_\ell(a, 1), a^\vee)$ .

The argument is an object  $a$ . The output is the isomorphism  $\text{IsomorphismFromLeftInternalHomIntoTensorUnitToLeftDualObject}_a : \underline{\text{Hom}}_\ell(a, 1) \rightarrow a^\vee$ .

### 1.5.31 UniversalPropertyOfLeftDual (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `UniversalPropertyOfLeftDual(t, a, alpha)` (operation)

**Returns:** a morphism in  $\text{Hom}(t, a^\vee)$ .

The arguments are two objects  $t, a$ , and a morphism  $\alpha : t \otimes a \rightarrow 1$ . The output is the morphism  $t \rightarrow a^\vee$  given by the universal property of  $a^\vee$ .

### 1.5.32 LeftClosedMonoidalLambdaIntroduction (for IsCapCategoryMorphism)

▷ `LeftClosedMonoidalLambdaIntroduction(alpha)` (attribute)

**Returns:** a morphism in  $\text{Hom}(1, \underline{\text{Hom}}_\ell(a, b))$ .

The argument is a morphism  $\alpha : a \rightarrow b$ . The output is the corresponding morphism  $1 \rightarrow \underline{\text{Hom}}_\ell(a, b)$  under the tensor hom adjunction.

### 1.5.33 LeftClosedMonoidalLambdaElimination (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `LeftClosedMonoidalLambdaElimination(a, b, alpha)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, b)$ .

The arguments are two objects  $a, b$ , and a morphism  $\alpha : 1 \rightarrow \underline{\text{Hom}}_\ell(a, b)$ . The output is a morphism  $a \rightarrow b$  corresponding to  $\alpha$  under the tensor hom adjunction.

### 1.5.34 IsomorphismFromObjectToLeftInternalHom (for IsCapCategoryObject)

▷ `IsomorphismFromObjectToLeftInternalHom(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{Hom}}_\ell(1, a))$ .

The argument is an object  $a$ . The output is the natural isomorphism  $a \rightarrow \underline{\text{Hom}}_\ell(1, a)$ .

### 1.5.35 IsomorphismFromObjectToLeftInternalHomWithGivenLeftInternalHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromObjectToLeftInternalHomWithGivenLeftInternalHom(a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, r)$ .

The argument is an object  $a$ , and an object  $r = \underline{\text{Hom}}_\ell(1, a)$ . The output is the natural isomorphism  $a \rightarrow \underline{\text{Hom}}_\ell(1, a)$ .

### 1.5.36 IsomorphismFromLeftInternalHomToObject (for IsCapCategoryObject)

▷ `IsomorphismFromLeftInternalHomToObject(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}_\ell(1, a), a)$ .

The argument is an object  $a$ . The output is the natural isomorphism  $\underline{\text{Hom}}_\ell(1, a) \rightarrow a$ .

### 1.5.37 IsomorphismFromLeftInternalHomToObjectWithGivenLeftInternalHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromLeftInternalHomToObjectWithGivenLeftInternalHom(a, s)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, a)$ .

The argument is an object  $a$ , and an object  $s = \underline{\text{Hom}}_\ell(1, a)$ . The output is the natural isomorphism  $\underline{\text{Hom}}_\ell(1, a) \rightarrow a$ .

## 1.6 Closed Monoidal Categories

A monoidal category  $\mathbf{C}$  which has for each functor  $- \otimes b : \mathbf{C} \rightarrow \mathbf{C}$  a right adjoint (denoted by  $\underline{\text{Hom}}_\ell(b, -)$ ) is called a *closed monoidal category*.

If no operations involving duals are installed manually, the dual objects will be derived as  $a^\vee := \underline{\text{Hom}}_\ell(a, 1)$ .

The corresponding GAP property is called `IsClosedMonoidalCategory`.

### 1.6.1 InternalHomOnObjects (for IsCapCategoryObject, IsCapCategoryObject)

▷ `InternalHomOnObjects(a, b)` (operation)

**Returns:** an object

The arguments are two objects  $a, b$ . The output is the internal hom object  $\underline{\text{Hom}}(a, b)$ .

### 1.6.2 InternalHomOnMorphisms (for IsCapCategoryMorphism, IsCapCategoryMorphism)

▷ `InternalHomOnMorphisms(alpha, beta)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a', b), \underline{\text{Hom}}(a, b'))$

The arguments are two morphisms  $\alpha : a \rightarrow a', \beta : b \rightarrow b'$ . The output is the internal hom morphism  $\underline{\text{Hom}}(\alpha, \beta) : \underline{\text{Hom}}(a', b) \rightarrow \underline{\text{Hom}}(a, b')$ .



### 1.6.3 InternalHomOnMorphismsWithGivenInternalHoms (for IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryMorphism, IsCapCategoryObject)

▷ InternalHomOnMorphismsWithGivenInternalHoms( $s$ ,  $\alpha$ ,  $\beta$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$

The arguments are an object  $s = \underline{\text{Hom}}(a', b)$ , two morphisms  $\alpha : a \rightarrow a'$ ,  $\beta : b \rightarrow b'$ , and an object  $r = \underline{\text{Hom}}(a, b')$ . The output is the internal hom morphism  $\underline{\text{Hom}}(\alpha, \beta) : \underline{\text{Hom}}(a', b) \rightarrow \underline{\text{Hom}}(a, b')$ .

### 1.6.4 ClosedMonoidalRightEvaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ ClosedMonoidalRightEvaluationMorphism( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes \underline{\text{Hom}}(a, b), b)$ .

The arguments are two objects  $a, b$ . The output is the right evaluation morphism  $\text{ev}_{a,b} : a \otimes \underline{\text{Hom}}(a, b) \rightarrow b$ , i.e., the counit of the tensor hom adjunction.

### 1.6.5 ClosedMonoidalRightEvaluationMorphismWithGivenSource (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ ClosedMonoidalRightEvaluationMorphismWithGivenSource( $a$ ,  $b$ ,  $s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, b)$ .

The arguments are two objects  $a, b$  and an object  $s = a \otimes \underline{\text{Hom}}(a, b)$ . The output is the right evaluation morphism  $\text{ev}_{a,b} : a \otimes \underline{\text{Hom}}(a, b) \rightarrow b$ , i.e., the counit of the tensor hom adjunction.

### 1.6.6 ClosedMonoidalRightCoevaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ ClosedMonoidalRightCoevaluationMorphism( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, \underline{\text{Hom}}(a, a \otimes b))$ .

The arguments are two objects  $a, b$ . The output is the right coevaluation morphism  $\text{coev}_{a,b} : b \rightarrow \underline{\text{Hom}}(a, a \otimes b)$ , i.e., the unit of the tensor hom adjunction.

### 1.6.7 ClosedMonoidalRightCoevaluationMorphismWithGivenRange (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ ClosedMonoidalRightCoevaluationMorphismWithGivenRange( $a$ ,  $b$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, r)$ .

The arguments are two objects  $a, b$  and an object  $r = \underline{\text{Hom}}(a, a \otimes b)$ . The output is the right coevaluation morphism  $\text{coev}_{a,b} : b \rightarrow \underline{\text{Hom}}(a, a \otimes b)$ , i.e., the unit of the tensor hom adjunction.

### 1.6.8 TensorProductToInternalHomRightAdjunctMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ TensorProductToInternalHomRightAdjunctMorphism( $a$ ,  $b$ ,  $f$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, \underline{\text{Hom}}(a, c))$ .

The arguments are two objects  $a, b$  and a morphism  $f : a \otimes b \rightarrow c$ . The output is a morphism  $g : b \rightarrow \underline{\text{Hom}}(a, c)$  corresponding to  $f$  under the tensor hom adjunction.

### 1.6.9 TensorProductToInternalHomRightAdjunctMorphismWithGivenInternalHom (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ `TensorProductToInternalHomRightAdjunctMorphismWithGivenInternalHom(a, b, f, i)` (operation)

**Returns:** a morphism in  $\text{Hom}(b, i)$ .

The arguments are two objects  $a, b$ , a morphism  $f : a \otimes b \rightarrow c$  and an object  $i = \underline{\text{Hom}}(a, c)$ . The output is a morphism  $g : b \rightarrow \underline{\text{Hom}}(a, c)$  corresponding to  $f$  under the tensor hom adjunction.

### 1.6.10 InternalHomToTensorProductRightAdjunctMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `InternalHomToTensorProductRightAdjunctMorphism(a, c, g)` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, c)$ .

The arguments are two objects  $a, c$  and a morphism  $g : b \rightarrow \underline{\text{Hom}}(a, c)$ . The output is a morphism  $f : a \otimes b \rightarrow c$  corresponding to  $g$  under the tensor hom adjunction.

### 1.6.11 InternalHomToTensorProductRightAdjunctMorphismWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ `InternalHomToTensorProductRightAdjunctMorphismWithGivenTensorProduct(a, c, g, s)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, c)$ .

The arguments are two objects  $a, c$ , a morphism  $g : b \rightarrow \underline{\text{Hom}}(a, c)$  and an object  $s = a \otimes b$ . The output is a morphism  $f : s \rightarrow c$  corresponding to  $g$  under the tensor hom adjunction.

### 1.6.12 ClosedMonoidalLeftEvaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ `ClosedMonoidalLeftEvaluationMorphism(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, b) \otimes a, b)$ .

The arguments are two objects  $a, b$ . The output is the left evaluation morphism  $\text{ev}_{a,b} : \underline{\text{Hom}}(a, b) \otimes a \rightarrow b$ , i.e., the counit of the tensor hom adjunction.

### 1.6.13 ClosedMonoidalLeftEvaluationMorphismWithGivenSource (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `ClosedMonoidalLeftEvaluationMorphismWithGivenSource(a, b, s)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, b)$ .

The arguments are two objects  $a, b$  and an object  $s = \underline{\text{Hom}}(a, b) \otimes a$ . The output is the left evaluation morphism  $\text{ev}_{a,b} : \underline{\text{Hom}}(a, b) \otimes a \rightarrow b$ , i.e., the counit of the tensor hom adjunction.

#### 1.6.14 ClosedMonoidalLeftCoevaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ ClosedMonoidalLeftCoevaluationMorphism( $a, b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, \underline{\text{Hom}}(a, b \otimes a))$ .

The arguments are two objects  $a, b$ . The output is the left coevaluation morphism  $\text{coev}_{a,b} : b \rightarrow \underline{\text{Hom}}(a, b \otimes a)$ , i.e., the unit of the tensor hom adjunction.

#### 1.6.15 ClosedMonoidalLeftCoevaluationMorphismWithGivenRange (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ ClosedMonoidalLeftCoevaluationMorphismWithGivenRange( $a, b, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, r)$ .

The arguments are two objects  $a, b$  and an object  $r = \underline{\text{Hom}}(a, b \otimes a)$ . The output is the left coevaluation morphism  $\text{coev}_{a,b} : b \rightarrow \underline{\text{Hom}}(a, b \otimes a)$ , i.e., the unit of the tensor hom adjunction.

#### 1.6.16 TensorProductToInternalHomLeftAdjunctMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ TensorProductToInternalHomLeftAdjunctMorphism( $a, b, f$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{Hom}}(b, c))$ .

The arguments are two objects  $a, b$  and a morphism  $f : a \otimes b \rightarrow c$ . The output is a morphism  $g : a \rightarrow \underline{\text{Hom}}(b, c)$  corresponding to  $f$  under the tensor hom adjunction.

#### 1.6.17 TensorProductToInternalHomLeftAdjunctMorphismWithGivenInternalHom (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ TensorProductToInternalHomLeftAdjunctMorphismWithGivenInternalHom( $a, b, f, i$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, i)$ .

The arguments are two objects  $a, b$ , a morphism  $f : a \otimes b \rightarrow c$  and an object  $i = \underline{\text{Hom}}(b, c)$ . The output is a morphism  $g : a \rightarrow \underline{\text{Hom}}(b, c)$  corresponding to  $f$  under the tensor hom adjunction.

#### 1.6.18 InternalHomToTensorProductLeftAdjunctMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ InternalHomToTensorProductLeftAdjunctMorphism( $b, c, g$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes b, c)$ .

The arguments are two objects  $b, c$  and a morphism  $g : a \rightarrow \underline{\text{Hom}}(b, c)$ . The output is a morphism  $f : a \otimes b \rightarrow c$  corresponding to  $g$  under the tensor hom adjunction.

#### 1.6.19 InternalHomToTensorProductLeftAdjunctMorphismWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ InternalHomToTensorProductLeftAdjunctMorphismWithGivenTensorProduct( $b, c, g, s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, c)$ .

The arguments are two objects  $b, c$ , a morphism  $g : a \rightarrow \underline{\text{Hom}}(b, c)$  and an object  $s = a \otimes b$ . The output is a morphism  $f : s \rightarrow c$  corresponding to  $g$  under the tensor hom adjunction.

### 1.6.20 MonoidalPreComposeMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MonoidalPreComposeMorphism( $a, b, c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, b) \otimes \underline{\text{Hom}}(b, c), \underline{\text{Hom}}(a, c))$ .

The arguments are three objects  $a, b, c$ . The output is the precomposition morphism  $\text{MonoidalPreComposeMorphism}_{a,b,c} : \underline{\text{Hom}}(a, b) \otimes \underline{\text{Hom}}(b, c) \rightarrow \underline{\text{Hom}}(a, c)$ .

### 1.6.21 MonoidalPreComposeMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MonoidalPreComposeMorphismWithGivenObjects( $s, a, b, c, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = \underline{\text{Hom}}(a, b) \otimes \underline{\text{Hom}}(b, c)$ , three objects  $a, b, c$ , and an object  $r = \underline{\text{Hom}}(a, c)$ . The output is the precomposition morphism  $\text{MonoidalPreComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{Hom}}(a, b) \otimes \underline{\text{Hom}}(b, c) \rightarrow \underline{\text{Hom}}(a, c)$ .

### 1.6.22 MonoidalPostComposeMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MonoidalPostComposeMorphism( $a, b, c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(b, c) \otimes \underline{\text{Hom}}(a, b), \underline{\text{Hom}}(a, c))$ .

The arguments are three objects  $a, b, c$ . The output is the postcomposition morphism  $\text{MonoidalPostComposeMorphism}_{a,b,c} : \underline{\text{Hom}}(b, c) \otimes \underline{\text{Hom}}(a, b) \rightarrow \underline{\text{Hom}}(a, c)$ .

### 1.6.23 MonoidalPostComposeMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MonoidalPostComposeMorphismWithGivenObjects( $s, a, b, c, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = \underline{\text{Hom}}(b, c) \otimes \underline{\text{Hom}}(a, b)$ , three objects  $a, b, c$ , and an object  $r = \underline{\text{Hom}}(a, c)$ . The output is the postcomposition morphism  $\text{MonoidalPostComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{Hom}}(b, c) \otimes \underline{\text{Hom}}(a, b) \rightarrow \underline{\text{Hom}}(a, c)$ .

### 1.6.24 DualOnObjects (for IsCapCategoryObject)

▷ DualOnObjects( $a$ ) (attribute)

**Returns:** an object

The argument is an object  $a$ . The output is its dual object  $a^\vee$ .

### 1.6.25 DualOnMorphisms (for IsCapCategoryMorphism)

▷ `DualOnMorphisms(alpha)` (attribute)

**Returns:** a morphism in  $\text{Hom}(b^\vee, a^\vee)$ .

The argument is a morphism  $\alpha : a \rightarrow b$ . The output is its dual morphism  $\alpha^\vee : b^\vee \rightarrow a^\vee$ .

### 1.6.26 DualOnMorphismsWithGivenDuals (for IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ `DualOnMorphismsWithGivenDuals(s, alpha, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The argument is an object  $s = b^\vee$ , a morphism  $\alpha : a \rightarrow b$ , and an object  $r = a^\vee$ . The output is the dual morphism  $\alpha^\vee : b^\vee \rightarrow a^\vee$ .

### 1.6.27 EvaluationForDual (for IsCapCategoryObject)

▷ `EvaluationForDual(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes a, 1)$ .

The argument is an object  $a$ . The output is the evaluation morphism  $\text{ev}_a : a^\vee \otimes a \rightarrow 1$ .

### 1.6.28 EvaluationForDualWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `EvaluationForDualWithGivenTensorProduct(s, a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = a^\vee \otimes a$ , an object  $a$ , and an object  $r = 1$ . The output is the evaluation morphism  $\text{ev}_a : a^\vee \otimes a \rightarrow 1$ .

### 1.6.29 MorphismToBidual (for IsCapCategoryObject)

▷ `MorphismToBidual(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a, (a^\vee)^\vee)$ .

The argument is an object  $a$ . The output is the morphism to the bidual  $a \rightarrow (a^\vee)^\vee$ .

### 1.6.30 MorphismToBidualWithGivenBidual (for IsCapCategoryObject, IsCapCategoryObject)

▷ `MorphismToBidualWithGivenBidual(a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, r)$ .

The arguments are an object  $a$ , and an object  $r = (a^\vee)^\vee$ . The output is the morphism to the bidual  $a \rightarrow (a^\vee)^\vee$ .

### 1.6.31 TensorProductInternalHomCompatibilityMorphism (for IsList)

▷ `TensorProductInternalHomCompatibilityMorphism(list)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b'), \underline{\text{Hom}}(a \otimes b, a' \otimes b'))$ .

The argument is a list of four objects  $[a, a', b, b']$ . The output is the natural morphism  $\text{TensorProductInternalHomCompatibilityMorphism}_{a, a', b, b'} : \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b') \rightarrow \underline{\text{Hom}}(a \otimes b, a' \otimes b')$ .

### 1.6.32 TensorProductInternalHomCompatibilityMorphismWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ `TensorProductInternalHomCompatibilityMorphismWithGivenObjects(s, list, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are a list of four objects  $[a, a', b, b']$ , and two objects  $s = \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b')$  and  $r = \underline{\text{Hom}}(a \otimes b, a' \otimes b')$ . The output is the natural morphism  $\text{TensorProductInternalHomCompatibilityMorphismWithGivenObjects}_{a, a', b, b'} : \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b') \rightarrow \underline{\text{Hom}}(a \otimes b, a' \otimes b')$ .

### 1.6.33 TensorProductDualityCompatibilityMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ `TensorProductDualityCompatibilityMorphism(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes b^\vee, (a \otimes b)^\vee)$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{TensorProductDualityCompatibilityMorphism} : a^\vee \otimes b^\vee \rightarrow (a \otimes b)^\vee$ .

### 1.6.34 TensorProductDualityCompatibilityMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `TensorProductDualityCompatibilityMorphismWithGivenObjects(s, a, b, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = a^\vee \otimes b^\vee$ , two objects  $a, b$ , and an object  $r = (a \otimes b)^\vee$ . The output is the natural morphism  $\text{TensorProductDualityCompatibilityMorphismWithGivenObjects}_{a, b} : a^\vee \otimes b^\vee \rightarrow (a \otimes b)^\vee$ .

### 1.6.35 MorphismFromTensorProductToInternalHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `MorphismFromTensorProductToInternalHom(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes b, \underline{\text{Hom}}(a, b))$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{MorphismFromTensorProductToInternalHom}_{a, b} : a^\vee \otimes b \rightarrow \underline{\text{Hom}}(a, b)$ .

### 1.6.36 MorphismFromTensorProductToInternalHomWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `MorphismFromTensorProductToInternalHomWithGivenObjects(s, a, b, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = a^\vee \otimes b$ , two objects  $a, b$ , and an object  $r = \underline{\text{Hom}}(a, b)$ . The output is the natural morphism  $\text{MorphismFromTensorProductToInternalHomWithGivenObjects}_{a, b} : a^\vee \otimes b \rightarrow \underline{\text{Hom}}(a, b)$ .

### 1.6.37 IsomorphismFromDualObjectToInternalHomIntoTensorUnit (for IsCapCategoryObject)

▷ `IsomorphismFromDualObjectToInternalHomIntoTensorUnit(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a^\vee, \underline{\text{Hom}}(a, 1))$ .

The argument is an object  $a$ . The output is the isomorphism  $\text{IsomorphismFromDualObjectToInternalHomIntoTensorUnit}_a : a^\vee \rightarrow \underline{\text{Hom}}(a, 1)$ .

### 1.6.38 IsomorphismFromInternalHomIntoTensorUnitToDualObject (for IsCapCategoryObject)

▷ `IsomorphismFromInternalHomIntoTensorUnitToDualObject(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, 1), a^\vee)$ .

The argument is an object  $a$ . The output is the isomorphism  $\text{IsomorphismFromInternalHomIntoTensorUnitToDualObject}_a : \underline{\text{Hom}}(a, 1) \rightarrow a^\vee$ .

### 1.6.39 UniversalPropertyOfDual (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `UniversalPropertyOfDual(t, a, alpha)` (operation)

**Returns:** a morphism in  $\text{Hom}(t, a^\vee)$ .

The arguments are two objects  $t, a$ , and a morphism  $\alpha : t \otimes a \rightarrow 1$ . The output is the morphism  $t \rightarrow a^\vee$  given by the universal property of  $a^\vee$ .

### 1.6.40 LambdaIntroduction (for IsCapCategoryMorphism)

▷ `LambdaIntroduction(alpha)` (attribute)

**Returns:** a morphism in  $\text{Hom}(1, \underline{\text{Hom}}(a, b))$ .

The argument is a morphism  $\alpha : a \rightarrow b$ . The output is the corresponding morphism  $1 \rightarrow \underline{\text{Hom}}(a, b)$  under the tensor hom adjunction.

### 1.6.41 LambdaElimination (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `LambdaElimination(a, b, alpha)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, b)$ .

The arguments are two objects  $a, b$ , and a morphism  $\alpha : 1 \rightarrow \underline{\text{Hom}}(a, b)$ . The output is a morphism  $a \rightarrow b$  corresponding to  $\alpha$  under the tensor hom adjunction.

### 1.6.42 IsomorphismFromObjectToInternalHom (for IsCapCategoryObject)

▷ `IsomorphismFromObjectToInternalHom(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{Hom}}(1, a))$ .

The argument is an object  $a$ . The output is the natural isomorphism  $a \rightarrow \underline{\text{Hom}}(1, a)$ .

### 1.6.43 IsomorphismFromObjectToInternalHomWithGivenInternalHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ IsomorphismFromObjectToInternalHomWithGivenInternalHom( $a, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, r)$ .

The argument is an object  $a$ , and an object  $r = \underline{\text{Hom}}(1, a)$ . The output is the natural isomorphism  $a \rightarrow \underline{\text{Hom}}(1, a)$ .

### 1.6.44 IsomorphismFromInternalHomToObject (for IsCapCategoryObject)

▷ IsomorphismFromInternalHomToObject( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(1, a), a)$ .

The argument is an object  $a$ . The output is the natural isomorphism  $\underline{\text{Hom}}(1, a) \rightarrow a$ .

### 1.6.45 IsomorphismFromInternalHomToObjectWithGivenInternalHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ IsomorphismFromInternalHomToObjectWithGivenInternalHom( $a, s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, a)$ .

The argument is an object  $a$ , and an object  $s = \underline{\text{Hom}}(1, a)$ . The output is the natural isomorphism  $\underline{\text{Hom}}(1, a) \rightarrow a$ .

## 1.7 Left Coclosed Monoidal Categories

A monoidal category  $\mathbf{C}$  which has for each functor  $- \otimes b : \mathbf{C} \rightarrow \mathbf{C}$  a left adjoint (denoted by  $\underline{\text{coHom}}(-, b)$ ) is called a *left coclosed monoidal category*.

If no operations involving left coduals are installed manually, the left codual objects will be derived as  $a_\vee := \underline{\text{coHom}}(1, a)$ .

The corresponding GAP property is called IsLeftCoclosedMonoidalCategory.

### 1.7.1 LeftInternalCoHomOnObjects (for IsCapCategoryObject, IsCapCategoryObject)

▷ LeftInternalCoHomOnObjects( $a, b$ ) (operation)

**Returns:** an object

The arguments are two objects  $a, b$ . The output is the internal cohom object  $\underline{\text{coHom}}_\ell(a, b)$ .

### 1.7.2 LeftInternalCoHomOnMorphisms (for IsCapCategoryMorphism, IsCapCategoryMorphism)

▷ LeftInternalCoHomOnMorphisms( $\alpha, \beta$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}_\ell(a, b'), \underline{\text{coHom}}_\ell(a', b))$

The arguments are two morphisms  $\alpha : a \rightarrow a', \beta : b \rightarrow b'$ . The output is the internal cohom morphism  $\underline{\text{coHom}}_\ell(\alpha, \beta) : \underline{\text{coHom}}_\ell(a, b') \rightarrow \underline{\text{coHom}}_\ell(a', b)$ .



### 1.7.3 LeftInternalCoHomOnMorphismsWithGivenLeftInternalCoHoms (for IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryMorphism, IsCapCategoryObject)

▷ LeftInternalCoHomOnMorphismsWithGivenLeftInternalCoHoms( $s$ ,  $\alpha$ ,  $\beta$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$

The arguments are an object  $s = \underline{\text{coHom}}_\ell(a, b')$ , two morphisms  $\alpha : a \rightarrow a'$ ,  $\beta : b \rightarrow b'$ , and an object  $r = \underline{\text{coHom}}_\ell(a', b)$ . The output is the internal cohomorphism  $\underline{\text{coHom}}_\ell(\alpha, \beta) : \underline{\text{coHom}}_\ell(a, b') \rightarrow \underline{\text{coHom}}_\ell(a', b)$ .

### 1.7.4 LeftCoclosedMonoidalEvaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ LeftCoclosedMonoidalEvaluationMorphism( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, \underline{\text{coHom}}_\ell(b, a) \otimes a)$ .

The arguments are two objects  $a, b$ . The output is the coclosed evaluation morphism  $\text{coclev}_{a,b} : b \rightarrow \underline{\text{coHom}}_\ell(b, a) \otimes a$ , i.e., the unit of the cohom tensor adjunction.

### 1.7.5 LeftCoclosedMonoidalEvaluationMorphismWithGivenRange (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftCoclosedMonoidalEvaluationMorphismWithGivenRange( $a$ ,  $b$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, r)$ .

The arguments are two objects  $a, b$  and an object  $r = \underline{\text{coHom}}_\ell(b, a) \otimes a$ . The output is the coclosed evaluation morphism  $\text{coclev}_{a,b} : b \rightarrow \underline{\text{coHom}}_\ell(b, a) \otimes a$ , i.e., the unit of the cohom tensor adjunction.

### 1.7.6 LeftCoclosedMonoidalCoevaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ LeftCoclosedMonoidalCoevaluationMorphism( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}_\ell(b \otimes a, a), b)$ .

The arguments are two objects  $a, b$ . The output is the coclosed coevaluation morphism  $\text{coclcoev}_{a,b} : \underline{\text{coHom}}_\ell(b \otimes a, a) \rightarrow b$ , i.e., the counit of the cohom tensor adjunction.

### 1.7.7 LeftCoclosedMonoidalCoevaluationMorphismWithGivenSource (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftCoclosedMonoidalCoevaluationMorphismWithGivenSource( $a$ ,  $b$ ,  $s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, b)$ .

The arguments are two objects  $a, b$  and an object  $s = \underline{\text{coHom}}_\ell(b \otimes a, a)$ . The output is the coclosed coevaluation morphism  $\text{coclcoev}_{a,b} : \underline{\text{coHom}}_\ell(b \otimes a, a) \rightarrow b$ , i.e., the unit of the cohom tensor adjunction.

### 1.7.8 TensorProductToLeftInternalCoHomAdjunctMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ TensorProductToLeftInternalCoHomAdjunctMorphism( $b, c, g$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}_\ell(a, c), b)$ .

The arguments are two objects  $b, c$  and a morphism  $g : a \rightarrow b \otimes c$ . The output is a morphism  $f : \text{coHom}_\ell(a, c) \rightarrow b$  corresponding to  $g$  under the cohom tensor adjunction.

### 1.7.9 TensorProductToLeftInternalCoHomAdjunctMorphismWithGivenLeftInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ TensorProductToLeftInternalCoHomAdjunctMorphismWithGivenLeftInternalCoHom( $b, c, g, i$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(i, b)$ .

The arguments are two objects  $b, c$ , a morphism  $g : a \rightarrow b \otimes c$  and an object  $i = \text{coHom}_\ell(a, c)$ . The output is a morphism  $f : \text{coHom}_\ell(a, c) \rightarrow b$  corresponding to  $g$  under the cohom tensor adjunction.

### 1.7.10 LeftInternalCoHomToTensorProductAdjunctMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ LeftInternalCoHomToTensorProductAdjunctMorphism( $a, c, f$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, b \otimes c)$ .

The arguments are two objects  $a, c$  and a morphism  $f : \text{coHom}_\ell(a, c) \rightarrow b$ . The output is a morphism  $g : a \rightarrow b \otimes c$  corresponding to  $f$  under the cohom tensor adjunction.

### 1.7.11 LeftInternalCoHomToTensorProductAdjunctMorphismWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ LeftInternalCoHomToTensorProductAdjunctMorphismWithGivenTensorProduct( $a, c, f, t$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, t)$ .

The arguments are two objects  $a, c$ , a morphism  $f : \text{coHom}_\ell(a, c) \rightarrow b$  and an object  $t = b \otimes c$ . The output is a morphism  $g : a \rightarrow b \otimes c$  corresponding to  $f$  under the cohom tensor adjunction.

### 1.7.12 LeftCoclosedMonoidalPreCoComposeMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftCoclosedMonoidalPreCoComposeMorphism( $a, b, c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}_\ell(a, c), \text{coHom}_\ell(b, c) \otimes \text{coHom}_\ell(a, b))$ .

The arguments are three objects  $a, b, c$ . The output is the precocomposition morphism  $\text{LeftCoclosedMonoidalPreCoComposeMorphism}_{a,b,c} : \text{coHom}_\ell(a, c) \rightarrow \text{coHom}_\ell(b, c) \otimes \text{coHom}_\ell(a, b)$ .

### 1.7.13 LeftCoclosedMonoidalPreCoComposeMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftCoclosedMonoidalPreCoComposeMorphismWithGivenObjects( $s, a, b, c, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = \underline{\text{coHom}}_\ell(a, c)$ , three objects  $a, b, c$ , and an object  $r = \underline{\text{coHom}}_\ell(a, b) \otimes \underline{\text{coHom}}_\ell(b, c)$ . The output is the precocomposition morphism  $\text{LeftCoclosedMonoidalPreCoComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{coHom}}_\ell(a, c) \rightarrow \underline{\text{coHom}}_\ell(b, c) \otimes \underline{\text{coHom}}_\ell(a, b)$ .

### 1.7.14 LeftCoclosedMonoidalPostCoComposeMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftCoclosedMonoidalPostCoComposeMorphism( $a, b, c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}_\ell(a, c), \underline{\text{coHom}}_\ell(a, b) \otimes \underline{\text{coHom}}_\ell(b, c))$ .

The arguments are three objects  $a, b, c$ . The output is the postcocomposition morphism  $\text{LeftCoclosedMonoidalPostCoComposeMorphism}_{a,b,c} : \underline{\text{coHom}}_\ell(a, c) \rightarrow \underline{\text{coHom}}_\ell(a, b) \otimes \underline{\text{coHom}}_\ell(b, c)$ .

### 1.7.15 LeftCoclosedMonoidalPostCoComposeMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftCoclosedMonoidalPostCoComposeMorphismWithGivenObjects( $s, a, b, c, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = \underline{\text{coHom}}_\ell(a, c)$ , three objects  $a, b, c$ , and an object  $r = \underline{\text{coHom}}_\ell(b, c) \otimes \underline{\text{coHom}}_\ell(a, b)$ . The output is the postcocomposition morphism  $\text{LeftCoclosedMonoidalPostCoComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{coHom}}_\ell(a, c) \rightarrow \underline{\text{coHom}}_\ell(a, b) \otimes \underline{\text{coHom}}_\ell(b, c)$ .

### 1.7.16 LeftCoDualOnObjects (for IsCapCategoryObject)

▷ LeftCoDualOnObjects( $a$ ) (attribute)

**Returns:** an object

The argument is an object  $a$ . The output is its codual object  $a_\vee$ .

### 1.7.17 LeftCoDualOnMorphisms (for IsCapCategoryMorphism)

▷ LeftCoDualOnMorphisms( $\alpha$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(b_\vee, a_\vee)$ .

The argument is a morphism  $\alpha : a \rightarrow b$ . The output is its codual morphism  $\alpha_\vee : b_\vee \rightarrow a_\vee$ .

### 1.7.18 LeftCoDualOnMorphismsWithGivenLeftCoDuals (for IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ LeftCoDualOnMorphismsWithGivenLeftCoDuals( $s$ ,  $\alpha$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The argument is an object  $s = b_\vee$ , a morphism  $\alpha : a \rightarrow b$ , and an object  $r = a_\vee$ . The output is the dual morphism  $\alpha_\vee : b^\vee \rightarrow a^\vee$ .

### 1.7.19 LeftCoclosedMonoidalEvaluationForLeftCoDual (for IsCapCategoryObject)

▷ LeftCoclosedMonoidalEvaluationForLeftCoDual( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(1, a_\vee \otimes a)$ .

The argument is an object  $a$ . The output is the coclosed evaluation morphism  $\text{coclev}_a : 1 \rightarrow a_\vee \otimes a$ .

### 1.7.20 LeftCoclosedMonoidalEvaluationForLeftCoDualWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftCoclosedMonoidalEvaluationForLeftCoDualWithGivenTensorProduct( $s$ ,  $a$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = 1$ , an object  $a$ , and an object  $r = a_\vee \otimes a$ . The output is the coclosed evaluation morphism  $\text{coclev}_a : 1 \rightarrow a_\vee \otimes a$ .

### 1.7.21 MorphismFromLeftCoBidual (for IsCapCategoryObject)

▷ MorphismFromLeftCoBidual( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}((a_\vee)_\vee, a)$ .

The argument is an object  $a$ . The output is the morphism from the cobidual  $(a_\vee)_\vee \rightarrow a$ .

### 1.7.22 MorphismFromLeftCoBidualWithGivenLeftCoBidual (for IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromLeftCoBidualWithGivenLeftCoBidual( $a$ ,  $s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, a)$ .

The arguments are an object  $a$ , and an object  $s = (a_\vee)_\vee$ . The output is the morphism from the cobidual  $(a_\vee)_\vee \rightarrow a$ .

### 1.7.23 LeftInternalCoHomTensorProductCompatibilityMorphism (for IsList)

▷ LeftInternalCoHomTensorProductCompatibilityMorphism( $list$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}_\ell(a \otimes a', b \otimes b'), \text{coHom}_\ell(a, b) \otimes \text{coHom}_\ell(a', b'))$ .

The argument is a list of four objects  $[a, a', b, b']$ . The output is the natural morphism  $\text{LeftInternalCoHomTensorProductCompatibilityMorphism}_{a, a', b, b'} : \text{coHom}_\ell(a \otimes a', b \otimes b') \rightarrow \text{coHom}_\ell(a, b) \otimes \text{coHom}_\ell(a', b')$ .

### 1.7.24 LeftInternalCoHomTensorProductCompatibilityMorphismWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ LeftInternalCoHomTensorProductCompatibilityMorphismWithGivenObjects( $s$ ,  $list$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are a list of four objects  $[a, a', b, b']$ , and two objects  $s = \text{coHom}_\ell(a \otimes a', b \otimes b')$  and  $r = \text{coHom}_\ell(a, b) \otimes \text{coHom}_\ell(a', b')$ . The output is the natural morphism  $\text{LeftInternalCoHomTensorProductCompatibilityMorphismWithGivenObjects}_{a, a', b, b'} : \text{coHom}_\ell(a \otimes a', b \otimes b') \rightarrow \text{coHom}_\ell(a, b) \otimes \text{coHom}_\ell(a', b')$ .

### 1.7.25 LeftCoDualityTensorProductCompatibilityMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ LeftCoDualityTensorProductCompatibilityMorphism( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}((a \otimes b)_\vee, a_\vee \otimes b_\vee)$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{LeftCoDualityTensorProductCompatibilityMorphism} : (a \otimes b)_\vee \rightarrow a_\vee \otimes b_\vee$ .

### 1.7.26 LeftCoDualityTensorProductCompatibilityMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ LeftCoDualityTensorProductCompatibilityMorphismWithGivenObjects( $s$ ,  $a$ ,  $b$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = (a \otimes b)_\vee$ , two objects  $a, b$ , and an object  $r = a_\vee \otimes b_\vee$ . The output is the natural morphism  $\text{LeftCoDualityTensorProductCompatibilityMorphismWithGivenObjects}_{a, b} : (a \otimes b)_\vee \rightarrow a_\vee \otimes b_\vee$ .

### 1.7.27 MorphismFromLeftInternalCoHomToTensorProduct (for IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromLeftInternalCoHomToTensorProduct( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}_\ell(a, b), b_\vee \otimes a)$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{MorphismFromLeftInternalCoHomToTensorProduct}_{a, b} : \text{coHom}_\ell(a, b) \rightarrow b_\vee \otimes a$ .

### 1.7.28 MorphismFromLeftInternalCoHomToTensorProductWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromLeftInternalCoHomToTensorProductWithGivenObjects( $s$ ,  $a$ ,  $b$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = \text{coHom}_\ell(a, b)$ , two objects  $a, b$ , and an object  $r = b_\vee \otimes a$ . The output is the natural morphism  $\text{MorphismFromLeftInternalCoHomToTensorProductWithGivenObjects}_{a, b} : \text{coHom}_\ell(a, b) \rightarrow a \otimes b_\vee$ .

### 1.7.29 IsomorphismFromLeftCoDualObjectToLeftInternalCoHomFromTensorUnit (for IsCapCategoryObject)

▷ `IsomorphismFromLeftCoDualObjectToLeftInternalCoHomFromTensorUnit(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a_{\vee}, \underline{\text{coHom}}_{\ell}(1, a))$ .

The argument is an object  $a$ . The output is the isomorphism  $\text{IsomorphismFromLeftCoDualObjectToLeftInternalCoHomFromTensorUnit}_a : a_{\vee} \rightarrow \underline{\text{coHom}}_{\ell}(1, a)$ .

### 1.7.30 IsomorphismFromLeftInternalCoHomFromTensorUnitToLeftCoDualObject (for IsCapCategoryObject)

▷ `IsomorphismFromLeftInternalCoHomFromTensorUnitToLeftCoDualObject(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}_{\ell}(1, a), a_{\vee})$ .

The argument is an object  $a$ . The output is the isomorphism  $\text{IsomorphismFromLeftInternalCoHomFromTensorUnitToLeftCoDualObject}_a : \underline{\text{coHom}}_{\ell}(1, a) \rightarrow a_{\vee}$ .

### 1.7.31 UniversalPropertyOfLeftCoDual (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `UniversalPropertyOfLeftCoDual(t, a, alpha)` (operation)

**Returns:** a morphism in  $\text{Hom}(a_{\vee}, t)$ .

The arguments are two objects  $t, a$ , and a morphism  $\alpha : 1 \rightarrow t \otimes a$ . The output is the morphism  $a_{\vee} \rightarrow t$  given by the universal property of  $a_{\vee}$ .

### 1.7.32 LeftCoclosedMonoidalLambdaIntroduction (for IsCapCategoryMorphism)

▷ `LeftCoclosedMonoidalLambdaIntroduction(alpha)` (attribute)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}_{\ell}(a, b), 1)$ .

The argument is a morphism  $\alpha : a \rightarrow b$ . The output is the corresponding morphism  $\underline{\text{coHom}}_{\ell}(a, b) \rightarrow 1$  under the cohom tensor adjunction.

### 1.7.33 LeftCoclosedMonoidalLambdaElimination (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `LeftCoclosedMonoidalLambdaElimination(a, b, alpha)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, b)$ .

The arguments are two objects  $a, b$ , and a morphism  $\alpha : \underline{\text{coHom}}_{\ell}(a, b) \rightarrow 1$ . The output is a morphism  $a \rightarrow b$  corresponding to  $\alpha$  under the cohom tensor adjunction.

### 1.7.34 IsomorphismFromObjectToLeftInternalCoHom (for IsCapCategoryObject)

▷ `IsomorphismFromObjectToLeftInternalCoHom(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{coHom}}_{\ell}(a, 1))$ .

The argument is an object  $a$ . The output is the natural isomorphism  $a \rightarrow \underline{\text{coHom}}_{\ell}(a, 1)$ .

### 1.7.35 IsomorphismFromObjectToLeftInternalCoHomWithGivenLeftInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromObjectToLeftInternalCoHomWithGivenLeftInternalCoHom(a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, r)$ .

The argument is an object  $a$ , and an object  $r = \underline{\text{coHom}}_\ell(a, 1)$ . The output is the natural isomorphism  $a \rightarrow \underline{\text{coHom}}_\ell(a, 1)$ .

### 1.7.36 IsomorphismFromLeftInternalCoHomToObject (for IsCapCategoryObject)

▷ `IsomorphismFromLeftInternalCoHomToObject(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}_\ell(a, 1), a)$ .

The argument is an object  $a$ . The output is the natural isomorphism  $\underline{\text{coHom}}_\ell(a, 1) \rightarrow a$ .

### 1.7.37 IsomorphismFromLeftInternalCoHomToObjectWithGivenLeftInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromLeftInternalCoHomToObjectWithGivenLeftInternalCoHom(a, s)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, a)$ .

The argument is an object  $a$ , and an object  $s = \underline{\text{coHom}}_\ell(a, 1)$ . The output is the natural isomorphism  $\underline{\text{coHom}}_\ell(a, 1) \rightarrow a$ .

## 1.8 Coclosed Monoidal Categories

A monoidal category  $\mathbf{C}$  which has for each functor  $- \otimes b : \mathbf{C} \rightarrow \mathbf{C}$  a left adjoint (denoted by  $\underline{\text{coHom}}(-, b)$ ) is called a *coclosed monoidal category*.

If no operations involving coduals are installed manually, the codual objects will be derived as  $a_\vee := \underline{\text{coHom}}(1, a)$ .

The corresponding GAP property is called `IsCoclosedMonoidalCategory`.

### 1.8.1 InternalCoHomOnObjects (for IsCapCategoryObject, IsCapCategoryObject)

▷ `InternalCoHomOnObjects(a, b)` (operation)

**Returns:** an object

The arguments are two objects  $a, b$ . The output is the internal cohom object  $\underline{\text{coHom}}(a, b)$ .

### 1.8.2 InternalCoHomOnMorphisms (for IsCapCategoryMorphism, IsCapCategoryMorphism)

▷ `InternalCoHomOnMorphisms(alpha, beta)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b'), \underline{\text{coHom}}(a', b))$

The arguments are two morphisms  $\alpha : a \rightarrow a', \beta : b \rightarrow b'$ . The output is the internal cohom morphism  $\underline{\text{coHom}}(\alpha, \beta) : \underline{\text{coHom}}(a, b') \rightarrow \underline{\text{coHom}}(a', b)$ .

### 1.8.3 InternalCoHomOnMorphismsWithGivenInternalCoHoms (for IsCapCategory-Object, IsCapCategoryMorphism, IsCapCategoryMorphism, IsCapCategory-Object)

▷ InternalCoHomOnMorphismsWithGivenInternalCoHoms( $s$ ,  $\alpha$ ,  $\beta$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$

The arguments are an object  $s = \underline{\text{coHom}}(a, b')$ , two morphisms  $\alpha : a \rightarrow a', \beta : b \rightarrow b'$ , and an object  $r = \underline{\text{coHom}}(a', b)$ . The output is the internal cohom morphism  $\underline{\text{coHom}}(\alpha, \beta) : \underline{\text{coHom}}(a, b') \rightarrow \underline{\text{coHom}}(a', b)$ .

### 1.8.4 CoclosedMonoidalRightEvaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedMonoidalRightEvaluationMorphism( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, a \otimes \underline{\text{coHom}}(b, a))$ .

The arguments are two objects  $a, b$ . The output is the coclosed right evaluation morphism  $\text{coclev}_{a,b} : b \rightarrow a \otimes \underline{\text{coHom}}(b, a)$ , i.e., the unit of the cohom tensor adjunction.

### 1.8.5 CoclosedMonoidalRightEvaluationMorphismWithGivenRange (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedMonoidalRightEvaluationMorphismWithGivenRange( $a$ ,  $b$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, r)$ .

The arguments are two objects  $a, b$  and an object  $r = a \otimes \underline{\text{coHom}}(b, a)$ . The output is the coclosed right evaluation morphism  $\text{coclev}_{a,b} : b \rightarrow a \otimes \underline{\text{coHom}}(b, a)$ , i.e., the unit of the cohom tensor adjunction.

### 1.8.6 CoclosedMonoidalRightCoevaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedMonoidalRightCoevaluationMorphism( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a \otimes b, a), b)$ .

The arguments are two objects  $a, b$ . The output is the coclosed right coevaluation morphism  $\text{coclcov}_{a,b} : \underline{\text{coHom}}(a \otimes b, a) \rightarrow b$ , i.e., the counit of the cohom tensor adjunction.

### 1.8.7 CoclosedMonoidalRightCoevaluationMorphismWithGivenSource (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedMonoidalRightCoevaluationMorphismWithGivenSource( $a$ ,  $b$ ,  $s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, b)$ .

The arguments are two objects  $a, b$  and an object  $s = \underline{\text{coHom}}(a \otimes b, a)$ . The output is the coclosed right coevaluation morphism  $\text{coclcov}_{a,b} : \underline{\text{coHom}}(a \otimes b, a) \rightarrow b$ , i.e., the unit of the cohom tensor adjunction.



### 1.8.8 TensorProductToInternalCoHomRightAdjunctMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ TensorProductToInternalCoHomRightAdjunctMorphism( $b, c, g$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b), c)$ .

The arguments are two objects  $b, c$  and a morphism  $g : a \rightarrow b \otimes c$ . The output is a morphism  $f : \underline{\text{coHom}}(a, b) \rightarrow c$  corresponding to  $g$  under the cohom tensor adjunction.

### 1.8.9 TensorProductToInternalCoHomRightAdjunctMorphismWithGivenInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ TensorProductToInternalCoHomRightAdjunctMorphismWithGivenInternalCoHom( $b, c, g, i$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(i, c)$ .

The arguments are two objects  $b, c$ , a morphism  $g : a \rightarrow b \otimes c$  and an object  $i = \underline{\text{coHom}}(a, b)$ . The output is a morphism  $f : \underline{\text{coHom}}(a, b) \rightarrow c$  corresponding to  $g$  under the cohom tensor adjunction.

### 1.8.10 InternalCoHomToTensorProductRightAdjunctMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ InternalCoHomToTensorProductRightAdjunctMorphism( $a, b, f$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, b \otimes c)$ .

The arguments are two objects  $a, b$  and a morphism  $f : \underline{\text{coHom}}(a, b) \rightarrow c$ . The output is a morphism  $g : a \rightarrow b \otimes c$  corresponding to  $f$  under the cohom tensor adjunction.

### 1.8.11 InternalCoHomToTensorProductRightAdjunctMorphismWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ InternalCoHomToTensorProductRightAdjunctMorphismWithGivenTensorProduct( $a, b, f, t$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, t)$ .

The arguments are two objects  $a, b$ , a morphism  $f : \underline{\text{coHom}}(a, b) \rightarrow c$  and an object  $t = b \otimes c$ . The output is a morphism  $g : a \rightarrow t$  corresponding to  $f$  under the cohom tensor adjunction.

### 1.8.12 CoclosedMonoidalLeftEvaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedMonoidalLeftEvaluationMorphism( $a, b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, \underline{\text{coHom}}(b, a) \otimes a)$ .

The arguments are two objects  $a, b$ . The output is the coclosed left evaluation morphism  $\text{coclev}_{a,b} : b \rightarrow \underline{\text{coHom}}(b, a) \otimes a$ , i.e., the unit of the cohom tensor adjunction.

### 1.8.13 CoclosedMonoidalLeftEvaluationMorphismWithGivenRange (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedMonoidalLeftEvaluationMorphismWithGivenRange( $a$ ,  $b$ ,  $r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(b, r)$ .

The arguments are two objects  $a, b$  and an object  $r = \underline{\text{coHom}}(b, a) \otimes a$ . The output is the coclosed left evaluation morphism  $\text{coclev}_{a,b} : b \rightarrow \underline{\text{coHom}}(b, a) \otimes a$ , i.e., the unit of the cohom tensor adjunction.

### 1.8.14 CoclosedMonoidalLeftCoevaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedMonoidalLeftCoevaluationMorphism( $a$ ,  $b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(b \otimes a, a), b)$ .

The arguments are two objects  $a, b$ . The output is the coclosed left coevaluation morphism  $\text{coclcov}_{a,b} : \underline{\text{coHom}}(b \otimes a, a) \rightarrow b$ , i.e., the counit of the cohom tensor adjunction.

### 1.8.15 CoclosedMonoidalLeftCoevaluationMorphismWithGivenSource (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ CoclosedMonoidalLeftCoevaluationMorphismWithGivenSource( $a$ ,  $b$ ,  $s$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, b)$ .

The arguments are two objects  $a, b$  and an object  $s = \underline{\text{coHom}}(b \otimes a, a)$ . The output is the coclosed left coevaluation morphism  $\text{coclcov}_{a,b} : \underline{\text{coHom}}(b \otimes a, a) \rightarrow b$ , i.e., the unit of the cohom tensor adjunction.

### 1.8.16 TensorProductToInternalCoHomLeftAdjunctMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ TensorProductToInternalCoHomLeftAdjunctMorphism( $b$ ,  $c$ ,  $g$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, c), b)$ .

The arguments are two objects  $b, c$  and a morphism  $g : a \rightarrow b \otimes c$ . The output is a morphism  $f : \underline{\text{coHom}}(a, c) \rightarrow b$  corresponding to  $g$  under the cohom tensor adjunction.

### 1.8.17 TensorProductToInternalCoHomLeftAdjunctMorphismWithGivenInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ TensorProductToInternalCoHomLeftAdjunctMorphismWithGivenInternalCoHom( $b$ ,  $c$ ,  $g$ ,  $i$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(i, b)$ .

The arguments are two objects  $b, c$ , a morphism  $g : a \rightarrow b \otimes c$  and an object  $i = \underline{\text{coHom}}(a, c)$ . The output is a morphism  $f : \underline{\text{coHom}}(a, c) \rightarrow b$  corresponding to  $g$  under the cohom tensor adjunction.

### 1.8.18 InternalCoHomToTensorProductLeftAdjunctMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ InternalCoHomToTensorProductLeftAdjunctMorphism( $a, c, f$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, b \otimes c)$ .

The arguments are two objects  $a, c$  and a morphism  $f : \underline{\text{coHom}}(a, c) \rightarrow b$ . The output is a morphism  $g : a \rightarrow b \otimes c$  corresponding to  $f$  under the cohom tensor adjunction.

### 1.8.19 InternalCoHomToTensorProductLeftAdjunctMorphismWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ InternalCoHomToTensorProductLeftAdjunctMorphismWithGivenTensorProduct( $a, c, f, t$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a, t)$ .

The arguments are two objects  $a, c$ , a morphism  $f : \underline{\text{coHom}}(a, c) \rightarrow b$  and an object  $t = b \otimes c$ . The output is a morphism  $g : a \rightarrow t$  corresponding to  $f$  under the cohom tensor adjunction.

### 1.8.20 MonoidalPreCoComposeMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MonoidalPreCoComposeMorphism( $a, b, c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, c), \underline{\text{coHom}}(b, c) \otimes \underline{\text{coHom}}(a, b))$ .

The arguments are three objects  $a, b, c$ . The output is the precocomposition morphism  $\text{MonoidalPreCoComposeMorphism}_{a,b,c} : \underline{\text{coHom}}(a, c) \rightarrow \underline{\text{coHom}}(b, c) \otimes \underline{\text{coHom}}(a, b)$ .

### 1.8.21 MonoidalPreCoComposeMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MonoidalPreCoComposeMorphismWithGivenObjects( $s, a, b, c, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = \underline{\text{coHom}}(a, c)$ , three objects  $a, b, c$ , and an object  $r = \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(b, c)$ . The output is the precocomposition morphism  $\text{MonoidalPreCoComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{coHom}}(a, c) \rightarrow \underline{\text{coHom}}(b, c) \otimes \underline{\text{coHom}}(a, b)$ .

### 1.8.22 MonoidalPostCoComposeMorphism (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MonoidalPostCoComposeMorphism( $a, b, c$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, c), \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(b, c))$ .

The arguments are three objects  $a, b, c$ . The output is the postcocomposition morphism  $\text{MonoidalPostCoComposeMorphism}_{a,b,c} : \underline{\text{coHom}}(a, c) \rightarrow \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(b, c)$ .

### 1.8.23 MonoidalPostCoComposeMorphismWithGivenObjects (for IsCapCategory-Object, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `MonoidalPostCoComposeMorphismWithGivenObjects(s, a, b, c, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = \underline{\text{coHom}}(a, c)$ , three objects  $a, b, c$ , and an object  $r = \underline{\text{coHom}}(b, c) \otimes \underline{\text{coHom}}(a, b)$ . The output is the postcomposition morphism  $\text{MonoidalPostCoComposeMorphismWithGivenObjects}_{a,b,c} : \underline{\text{coHom}}(a, c) \rightarrow \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(b, c)$ .

### 1.8.24 CoDualOnObjects (for IsCapCategoryObject)

▷ `CoDualOnObjects(a)` (attribute)

**Returns:** an object

The argument is an object  $a$ . The output is its codual object  $a_\vee$ .

### 1.8.25 CoDualOnMorphisms (for IsCapCategoryMorphism)

▷ `CoDualOnMorphisms(alpha)` (attribute)

**Returns:** a morphism in  $\text{Hom}(b_\vee, a_\vee)$ .

The argument is a morphism  $\alpha : a \rightarrow b$ . The output is its codual morphism  $\alpha_\vee : b_\vee \rightarrow a_\vee$ .

### 1.8.26 CoDualOnMorphismsWithGivenCoDuals (for IsCapCategoryObject, IsCapCategoryMorphism, IsCapCategoryObject)

▷ `CoDualOnMorphismsWithGivenCoDuals(s, alpha, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The argument is an object  $s = b_\vee$ , a morphism  $\alpha : a \rightarrow b$ , and an object  $r = a_\vee$ . The output is the dual morphism  $\alpha_\vee : b^\vee \rightarrow a^\vee$ .

### 1.8.27 CoclosedEvaluationForCoDual (for IsCapCategoryObject)

▷ `CoclosedEvaluationForCoDual(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(1, a_\vee \otimes a)$ .

The argument is an object  $a$ . The output is the coclosed evaluation morphism  $\text{coclev}_a : 1 \rightarrow a_\vee \otimes a$ .

### 1.8.28 CoclosedEvaluationForCoDualWithGivenTensorProduct (for IsCapCategory-Object, IsCapCategoryObject, IsCapCategoryObject)

▷ `CoclosedEvaluationForCoDualWithGivenTensorProduct(s, a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = 1$ , an object  $a$ , and an object  $r = a_\vee \otimes a$ . The output is the coclosed evaluation morphism  $\text{coclev}_a : 1 \rightarrow a_\vee \otimes a$ .

### 1.8.29 MorphismFromCoBidual (for IsCapCategoryObject)

▷ `MorphismFromCoBidual(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}((a_V)_V, a)$ .

The argument is an object  $a$ . The output is the morphism from the cobidual  $(a_V)_V \rightarrow a$ .

### 1.8.30 MorphismFromCoBidualWithGivenCoBidual (for IsCapCategoryObject, IsCapCategoryObject)

▷ `MorphismFromCoBidualWithGivenCoBidual(a, s)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, a)$ .

The arguments are an object  $a$ , and an object  $s = (a_V)_V$ . The output is the morphism from the cobidual  $(a_V)_V \rightarrow a$ .

### 1.8.31 InternalCoHomTensorProductCompatibilityMorphism (for IsList)

▷ `InternalCoHomTensorProductCompatibilityMorphism(list)` (operation)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}(a \otimes a', b \otimes b'), \text{coHom}(a, b) \otimes \text{coHom}(a', b'))$ .

The argument is a list of four objects  $[a, a', b, b']$ . The output is the natural morphism  $\text{InternalCoHomTensorProductCompatibilityMorphism}_{a, a', b, b'} : \text{coHom}(a \otimes a', b \otimes b') \rightarrow \text{coHom}(a, b) \otimes \text{coHom}(a', b')$ .

### 1.8.32 InternalCoHomTensorProductCompatibilityMorphismWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ `InternalCoHomTensorProductCompatibilityMorphismWithGivenObjects(s, list, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are a list of four objects  $[a, a', b, b']$ , and two objects  $s = \text{coHom}(a \otimes a', b \otimes b')$  and  $r = \text{coHom}(a, b) \otimes \text{coHom}(a', b')$ . The output is the natural morphism  $\text{InternalCoHomTensorProductCompatibilityMorphismWithGivenObjects}_{a, a', b, b'} : \text{coHom}(a \otimes a', b \otimes b') \rightarrow \text{coHom}(a, b) \otimes \text{coHom}(a', b')$ .

### 1.8.33 CoDualityTensorProductCompatibilityMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ `CoDualityTensorProductCompatibilityMorphism(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}((a \otimes b)_V, a_V \otimes b_V)$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{CoDualityTensorProductCompatibilityMorphism} : (a \otimes b)_V \rightarrow a_V \otimes b_V$ .

### 1.8.34 CoDualityTensorProductCompatibilityMorphismWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `CoDualityTensorProductCompatibilityMorphismWithGivenObjects(s, a, b, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = (a \otimes b)_V$ , two objects  $a, b$ , and an object  $r = a_V \otimes b_V$ . The output is the natural morphism  $\text{CoDualityTensorProductCompatibilityMorphismWithGivenObjects}_{a,b} : (a \otimes b)_V \rightarrow a_V \otimes b_V$ .

### 1.8.35 MorphismFromInternalCoHomToTensorProduct (for IsCapCategoryObject, IsCapCategoryObject)

▷  $\text{MorphismFromInternalCoHomToTensorProduct}(a, b)$  (operation)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}(a, b), b_V \otimes a)$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{MorphismFromInternalCoHomToTensorProduct}_{a,b} : \text{coHom}(a, b) \rightarrow b_V \otimes a$ .

### 1.8.36 MorphismFromInternalCoHomToTensorProductWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷  $\text{MorphismFromInternalCoHomToTensorProductWithGivenObjects}(s, a, b, r)$  (operation)

**Returns:** a morphism in  $\text{Hom}(s, r)$ .

The arguments are an object  $s = \text{coHom}(a, b)$ , two objects  $a, b$ , and an object  $r = b_V \otimes a$ . The output is the natural morphism  $\text{MorphismFromInternalCoHomToTensorProductWithGivenObjects}_{a,b} : \text{coHom}(a, b) \rightarrow a \otimes b_V$ .

### 1.8.37 IsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit (for IsCapCategoryObject)

▷  $\text{IsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit}(a)$  (attribute)

**Returns:** a morphism in  $\text{Hom}(a_V, \text{coHom}(1, a))$ .

The argument is an object  $a$ . The output is the isomorphism  $\text{IsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit}_a : a_V \rightarrow \text{coHom}(1, a)$ .

### 1.8.38 IsomorphismFromInternalCoHomFromTensorUnitToCoDualObject (for IsCapCategoryObject)

▷  $\text{IsomorphismFromInternalCoHomFromTensorUnitToCoDualObject}(a)$  (attribute)

**Returns:** a morphism in  $\text{Hom}(\text{coHom}(1, a), a_V)$ .

The argument is an object  $a$ . The output is the isomorphism  $\text{IsomorphismFromInternalCoHomFromTensorUnitToCoDualObject}_a : \text{coHom}(1, a) \rightarrow a_V$ .

### 1.8.39 UniversalPropertyOfCoDual (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷  $\text{UniversalPropertyOfCoDual}(t, a, \alpha)$  (operation)

**Returns:** a morphism in  $\text{Hom}(a_V, t)$ .

The arguments are two objects  $t, a$ , and a morphism  $\alpha : 1 \rightarrow t \otimes a$ . The output is the morphism  $a_V \rightarrow t$  given by the universal property of  $a_V$ .

### 1.8.40 CoLambdaIntroduction (for IsCapCategoryMorphism)

▷ `CoLambdaIntroduction(alpha)` (attribute)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b), 1)$ .

The argument is a morphism  $\alpha : a \rightarrow b$ . The output is the corresponding morphism  $\underline{\text{coHom}}(a, b) \rightarrow 1$  under the cohom tensor adjunction.

### 1.8.41 CoLambdaElimination (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryMorphism)

▷ `CoLambdaElimination(a, b, alpha)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, b)$ .

The arguments are two objects  $a, b$ , and a morphism  $\alpha : \underline{\text{coHom}}(a, b) \rightarrow 1$ . The output is a morphism  $a \rightarrow b$  corresponding to  $\alpha$  under the cohom tensor adjunction.

### 1.8.42 IsomorphismFromObjectToInternalCoHom (for IsCapCategoryObject)

▷ `IsomorphismFromObjectToInternalCoHom(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a, \underline{\text{coHom}}(a, 1))$ .

The argument is an object  $a$ . The output is the natural isomorphism  $a \rightarrow \underline{\text{coHom}}(a, 1)$ .

### 1.8.43 IsomorphismFromObjectToInternalCoHomWithGivenInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromObjectToInternalCoHomWithGivenInternalCoHom(a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, r)$ .

The argument is an object  $a$ , and an object  $r = \underline{\text{coHom}}(a, 1)$ . The output is the natural isomorphism  $a \rightarrow \underline{\text{coHom}}(a, 1)$ .

### 1.8.44 IsomorphismFromInternalCoHomToObject (for IsCapCategoryObject)

▷ `IsomorphismFromInternalCoHomToObject(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, 1), a)$ .

The argument is an object  $a$ . The output is the natural isomorphism  $\underline{\text{coHom}}(a, 1) \rightarrow a$ .

### 1.8.45 IsomorphismFromInternalCoHomToObjectWithGivenInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromInternalCoHomToObjectWithGivenInternalCoHom(a, s)` (operation)

**Returns:** a morphism in  $\text{Hom}(s, a)$ .

The argument is an object  $a$ , and an object  $s = \underline{\text{coHom}}(a, 1)$ . The output is the natural isomorphism  $\underline{\text{coHom}}(a, 1) \rightarrow a$ .

## 1.9 Symmetric Closed Monoidal Categories

A monoidal category  $\mathbf{C}$  which is symmetric and closed is called a *symmetric closed monoidal category*.

The corresponding GAP property is given by `IsSymmetricClosedMonoidalCategory`.

## 1.10 Symmetric Coclosed Monoidal Categories

A monoidal category  $\mathbf{C}$  which is symmetric and coclosed is called a *symmetric coclosed monoidal category*.

The corresponding GAP property is given by `IsSymmetricCoclosedMonoidalCategory`.

## 1.11 Rigid Symmetric Closed Monoidal Categories

A symmetric closed monoidal category  $\mathbf{C}$  satisfying

- the natural morphism

$\underline{\text{Hom}}_\ell(a, a') \otimes \underline{\text{Hom}}_\ell(b, b') \rightarrow \underline{\text{Hom}}_\ell(a \otimes b, a' \otimes b')$  is an isomorphism,

- the natural morphism

$a \rightarrow \underline{\text{Hom}}_\ell(\underline{\text{Hom}}_\ell(a, 1), 1)$  is an isomorphism is called a *rigid symmetric closed monoidal category*.

If no operations involving the closed structure are installed manually, the internal hom objects will be derived as  $\underline{\text{Hom}}_\ell(a, b) := a^\vee \otimes b$  and, in particular,  $\underline{\text{Hom}}_\ell(a, 1) := a^\vee \otimes 1$ .

The corresponding GAP property is given by `IsRigidSymmetricClosedMonoidalCategory`.

### 1.11.1 IsomorphismFromTensorProductWithDualObjectToInternalHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromTensorProductWithDualObjectToInternalHom(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(a^\vee \otimes b, \underline{\text{Hom}}(a, b))$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{IsomorphismFromTensorProductWithDualObjectToInternalHom}_{a,b} : a^\vee \otimes b \rightarrow \underline{\text{Hom}}(a, b)$ .

### 1.11.2 IsomorphismFromInternalHomToTensorProductWithDualObject (for IsCapCategoryObject, IsCapCategoryObject)

▷ `IsomorphismFromInternalHomToTensorProductWithDualObject(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, b), a^\vee \otimes b)$ .

The arguments are two objects  $a, b$ . The output is the inverse of `IsomorphismFromTensorProductWithDualObjectToInternalHom`, namely  $\text{IsomorphismFromInternalHomToTensorProductWithDualObject}_{a,b} : \underline{\text{Hom}}(a, b) \rightarrow a^\vee \otimes b$ .

### 1.11.3 MorphismFromInternalHomToTensorProduct (for IsCapCategoryObject, IsCapCategoryObject)

▷ `MorphismFromInternalHomToTensorProduct(a, b)` (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, b), a^\vee \otimes b)$ .

The arguments are two objects  $a, b$ . The output is the inverse of `MorphismFromTensorProductToInternalHomWithGivenObjects`, namely  $\text{MorphismFromInternalHomToTensorProductWithGivenObjects}_{a,b} : \underline{\text{Hom}}(a, b) \rightarrow a^\vee \otimes b$ .



#### 1.11.4 MorphismFromInternalHomToTensorProductWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromInternalHomToTensorProductWithGivenObjects( $s, a, b, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a, b), a^\vee \otimes b)$ .

The arguments are an object  $s = \underline{\text{Hom}}(a, b)$ , two objects  $a, b$ , and an object  $r = a^\vee \otimes b$ . The output is the inverse of MorphismFromTensorProductToInternalHomWithGivenObjects, namely MorphismFromInternalHomToTensorProductWithGivenObjects $_{a,b} : \underline{\text{Hom}}(a, b) \rightarrow a^\vee \otimes b$ .

#### 1.11.5 TensorProductInternalHomCompatibilityMorphismInverse (for IsList)

▷ TensorProductInternalHomCompatibilityMorphismInverse( $list$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a \otimes b, a' \otimes b'), \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b'))$ .

The argument is a list of four objects  $[a, a', b, b']$ . The output is the natural morphism TensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects $_{a,a',b,b'} : \underline{\text{Hom}}(a \otimes b, a' \otimes b') \rightarrow \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b')$ .

#### 1.11.6 TensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ TensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects( $s, list, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{Hom}}(a \otimes b, a' \otimes b'), \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b'))$ .

The arguments are a list of four objects  $[a, a', b, b']$ , and two objects  $s = \underline{\text{Hom}}(a \otimes b, a' \otimes b')$  and  $r = \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b')$ . The output is the natural morphism TensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects $_{a,a',b,b'} : \underline{\text{Hom}}(a \otimes b, a' \otimes b') \rightarrow \underline{\text{Hom}}(a, a') \otimes \underline{\text{Hom}}(b, b')$ .

#### 1.11.7 CoevaluationForDual (for IsCapCategoryObject)

▷ CoevaluationForDual( $a$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(1, a \otimes a^\vee)$ .

The argument is an object  $a$ . The output is the coevaluation morphism  $\text{coev}_a : 1 \rightarrow a \otimes a^\vee$ .

#### 1.11.8 CoevaluationForDualWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ CoevaluationForDualWithGivenTensorProduct( $s, a, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(1, a \otimes a^\vee)$ .

The arguments are an object  $s = 1$ , an object  $a$ , and an object  $r = a \otimes a^\vee$ . The output is the coevaluation morphism  $\text{coev}_a : 1 \rightarrow a \otimes a^\vee$ .

#### 1.11.9 TraceMap (for IsCapCategoryMorphism)

▷ TraceMap( $alpha$ ) (attribute)

**Returns:** a morphism in  $\text{Hom}(1, 1)$ .

The argument is an endomorphism  $\alpha : a \rightarrow a$ . The output is the trace morphism  $\text{trace}_\alpha : 1 \rightarrow 1$ .

### 1.11.10 RankMorphism (for IsCapCategoryObject)

▷ RankMorphism(a) (attribute)

**Returns:** a morphism in  $\text{Hom}(1, 1)$ .

The argument is an object  $a$ . The output is the rank morphism  $\text{rank}_a : 1 \rightarrow 1$ .

### 1.11.11 MorphismFromBidual (for IsCapCategoryObject)

▷ MorphismFromBidual(a) (attribute)

**Returns:** a morphism in  $\text{Hom}((a^\vee)^\vee, a)$ .

The argument is an object  $a$ . The output is the inverse of the morphism to the bidual  $(a^\vee)^\vee \rightarrow a$ .

### 1.11.12 MorphismFromBidualWithGivenBidual (for IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromBidualWithGivenBidual(a, s) (operation)

**Returns:** a morphism in  $\text{Hom}((a^\vee)^\vee, a)$ .

The argument is an object  $a$ , and an object  $s = (a^\vee)^\vee$ . The output is the inverse of the morphism to the bidual  $(a^\vee)^\vee \rightarrow a$ .

## 1.12 Rigid Symmetric Coclosed Monoidal Categories

A symmetric coclosed monoidal category  $\mathbf{C}$  satisfying

- the natural morphism

$\underline{\text{coHom}}(a \otimes a', b \otimes b') \rightarrow \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(a', b')$  is an isomorphism,

- the natural morphism

$\underline{\text{coHom}}(1, \underline{\text{coHom}}(1, a)) \rightarrow a$  is an isomorphism is called a *rigid symmetric coclosed monoidal category*.

If no operations involving the coclosed structure are installed manually, the internal cohom objects will be derived as  $\underline{\text{coHom}}(a, b) := a \otimes b_\vee$  and, in particular,  $\underline{\text{coHom}}(1, a) := 1 \otimes a_\vee$ .

The corresponding GAP property is given by IsRigidSymmetricCoclosedMonoidalCategory.

### 1.12.1 IsomorphismFromInternalCoHomToTensorProductWithCoDualObject (for IsCapCategoryObject, IsCapCategoryObject)

▷ IsomorphismFromInternalCoHomToTensorProductWithCoDualObject(a, b) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b), b_\vee \otimes a)$ .

The arguments are two objects  $a, b$ . The output is the natural morphism  $\text{IsomorphismFromInternalCoHomToTensorProductWithCoDualObjectWithGivenObjects}_{a,b} : \underline{\text{coHom}}(a, b) \rightarrow b_\vee \otimes a$ .

### 1.12.2 IsomorphismFromTensorProductWithCoDualObjectToInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ IsomorphismFromTensorProductWithCoDualObjectToInternalCoHom( $a, b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a_{\vee} \otimes b, \underline{\text{coHom}}(b, a))$ .

The arguments are two objects  $a, b$ . The output is the inverse of IsomorphismFromInternalCoHomToTensorProductWithCoDualObject, namely IsomorphismFromTensorProductWithCoDualObjectToInternalCoHom $_{a,b} : a_{\vee} \otimes b \rightarrow \underline{\text{coHom}}(b, a)$ .

### 1.12.3 MorphismFromTensorProductToInternalCoHom (for IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromTensorProductToInternalCoHom( $a, b$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a_{\vee} \otimes b, \underline{\text{coHom}}(b, a))$ .

The arguments are two objects  $a, b$ . The output is the inverse of MorphismFromInternalCoHomToTensorProductWithGivenObjects, namely MorphismFromTensorProductToInternalCoHomWithGivenObjects $_{a,b} : a_{\vee} \otimes b \rightarrow \underline{\text{coHom}}(b, a)$ .

### 1.12.4 MorphismFromTensorProductToInternalCoHomWithGivenObjects (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ MorphismFromTensorProductToInternalCoHomWithGivenObjects( $s, a, b, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(a_{\vee} \otimes b, \underline{\text{coHom}}(b, a))$ .

The arguments are an object  $s_{\vee} = a \otimes b$ , two objects  $a, b$ , and an object  $r = \underline{\text{coHom}}(b, a)$ . The output is the inverse of MorphismFromInternalCoHomToTensorProductWithGivenObjects, namely MorphismFromTensorProductToInternalCoHomWithGivenObjects $_{a,b} : a_{\vee} \otimes b \rightarrow \underline{\text{coHom}}(b, a)$ .

### 1.12.5 InternalCoHomTensorProductCompatibilityMorphismInverse (for IsList)

▷ InternalCoHomTensorProductCompatibilityMorphismInverse( $list$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(a', b'), \underline{\text{coHom}}(a \otimes a', b \otimes b'))$ .

The argument is a list of four objects  $[a, a', b, b']$ . The output is the natural morphism InternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects $_{a,a',b,b'} : \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(a', b') \rightarrow \underline{\text{coHom}}(a \otimes a', b \otimes b')$ .

### 1.12.6 InternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects (for IsCapCategoryObject, IsList, IsCapCategoryObject)

▷ InternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects( $s, list, r$ ) (operation)

**Returns:** a morphism in  $\text{Hom}(\underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(a', b'), \underline{\text{coHom}}(a \otimes a', b \otimes b'))$ .

The arguments are a list of four objects  $[a, a', b, b']$ , and two objects  $s = \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(a', b')$  and  $r = \underline{\text{coHom}}(a \otimes a', b \otimes b')$ . The output is the natural morphism InternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects $_{a,a',b,b'} : \underline{\text{coHom}}(a, b) \otimes \underline{\text{coHom}}(a', b') \rightarrow \underline{\text{coHom}}(a \otimes a', b \otimes b')$ .

### 1.12.7 CoclosedCoevaluationForCoDual (for IsCapCategoryObject)

▷ `CoclosedCoevaluationForCoDual(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a \otimes a_V, 1)$ .

The argument is an object  $a$ . The output is the coclosed coevaluation morphism  $\text{coclcoev}_a : a \otimes a_V \rightarrow 1$ .

### 1.12.8 CoclosedCoevaluationForCoDualWithGivenTensorProduct (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ `CoclosedCoevaluationForCoDualWithGivenTensorProduct(s, a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a \otimes a_V, 1)$ .

The arguments are an object  $s = a \otimes a_V$ , an object  $a$ , and an object  $r = 1$ . The output is the coclosed coevaluation morphism  $\text{coclcoev}_a : a \otimes a_V \rightarrow 1$ .

### 1.12.9 CoTraceMap (for IsCapCategoryMorphism)

▷ `CoTraceMap(alpha)` (attribute)

**Returns:** a morphism in  $\text{Hom}(1, 1)$ .

The argument is an endomorphism  $\alpha : a \rightarrow a$ . The output is the cotrace morphism  $\text{cotrace}_\alpha : 1 \rightarrow 1$ .

### 1.12.10 CoRankMorphism (for IsCapCategoryObject)

▷ `CoRankMorphism(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(1, 1)$ .

The argument is an object  $a$ . The output is the corank morphism  $\text{corank}_a : 1 \rightarrow 1$ .

### 1.12.11 MorphismToCoBidual (for IsCapCategoryObject)

▷ `MorphismToCoBidual(a)` (attribute)

**Returns:** a morphism in  $\text{Hom}(a, (a_V)_V)$ .

The argument is an object  $a$ . The output is the inverse of the morphism from the cobidual  $a \rightarrow (a_V)_V$ .

### 1.12.12 MorphismToCoBidualWithGivenCoBidual (for IsCapCategoryObject, IsCapCategoryObject)

▷ `MorphismToCoBidualWithGivenCoBidual(a, r)` (operation)

**Returns:** a morphism in  $\text{Hom}(a, (a_V)_V)$ .

The argument is an object  $a$ , and an object  $r = (a_V)_V$ . The output is the inverse of the morphism from the cobidual  $a \rightarrow (a_V)_V$ .

## 1.13 Convenience Methods

### 1.13.1 InternalHom (for IsCapCategoryCell, IsCapCategoryCell)

▷ InternalHom( $a, b$ ) (operation)  
**Returns:** a cell

This is a convenience method. The arguments are two cells  $a, b$ . The output is the internal hom cell. If  $a, b$  are two CAP objects the output is the internal Hom object  $\underline{\text{Hom}}(a, b)$ . If at least one of the arguments is a CAP morphism the output is a CAP morphism, namely the internal hom on morphisms, where any object is replaced by its identity morphism.

### 1.13.2 InternalCoHom (for IsCapCategoryCell, IsCapCategoryCell)

▷ InternalCoHom( $a, b$ ) (operation)  
**Returns:** a cell

This is a convenience method. The arguments are two cells  $a, b$ . The output is the internal cohom cell. If  $a, b$  are two CAP objects the output is the internal cohom object  $\underline{\text{coHom}}(a, b)$ . If at least one of the arguments is a CAP morphism the output is a CAP morphism, namely the internal cohom on morphisms, where any object is replaced by its identity morphism.

### 1.13.3 LeftInternalHom (for IsCapCategoryCell, IsCapCategoryCell)

▷ LeftInternalHom( $a, b$ ) (operation)  
**Returns:** a cell

This is a convenience method. The arguments are two cells  $a, b$ . The output is the internal hom cell. If  $a, b$  are two CAP objects the output is the internal Hom object  $\underline{\text{Hom}}_\ell(a, b)$ . If at least one of the arguments is a CAP morphism the output is a CAP morphism, namely the internal hom on morphisms, where any object is replaced by its identity morphism.

### 1.13.4 LeftInternalCoHom (for IsCapCategoryCell, IsCapCategoryCell)

▷ LeftInternalCoHom( $a, b$ ) (operation)  
**Returns:** a cell

This is a convenience method. The arguments are two cells  $a, b$ . The output is the internal cohom cell. If  $a, b$  are two CAP objects the output is the internal cohom object  $\underline{\text{coHom}}_\ell(a, b)$ . If at least one of the arguments is a CAP morphism the output is a CAP morphism, namely the internal cohom on morphisms, where any object is replaced by its identity morphism.

## 1.14 Add-methods

### 1.14.1 AddLeftDistributivityExpanding (for IsCapCategory, IsFunction)

▷ AddLeftDistributivityExpanding( $C, F$ ) (operation)  
**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftDistributivityExpanding.  $F : (a, L) \mapsto \text{LeftDistributivityExpanding}(a, L)$ .

### 1.14.2 AddLeftDistributivityExpandingWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddLeftDistributivityExpandingWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftDistributivityExpandingWithGivenObjects.  $F : (s, a, L, r) \mapsto \text{LeftDistributivityExpandingWithGivenObjects}(s, a, L, r)$ .

### 1.14.3 AddLeftDistributivityFactoring (for IsCapCategory, IsFunction)

▷ AddLeftDistributivityFactoring( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftDistributivityFactoring.  $F : (a, L) \mapsto \text{LeftDistributivityFactoring}(a, L)$ .

### 1.14.4 AddLeftDistributivityFactoringWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddLeftDistributivityFactoringWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftDistributivityFactoringWithGivenObjects.  $F : (s, a, L, r) \mapsto \text{LeftDistributivityFactoringWithGivenObjects}(s, a, L, r)$ .

### 1.14.5 AddRightDistributivityExpanding (for IsCapCategory, IsFunction)

▷ AddRightDistributivityExpanding( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation RightDistributivityExpanding.  $F : (L, a) \mapsto \text{RightDistributivityExpanding}(L, a)$ .

### 1.14.6 AddRightDistributivityExpandingWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddRightDistributivityExpandingWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation RightDistributivityExpandingWithGivenObjects.  $F : (s, L, a, r) \mapsto \text{RightDistributivityExpandingWithGivenObjects}(s, L, a, r)$ .

### 1.14.7 AddRightDistributivityFactoring (for IsCapCategory, IsFunction)

▷ AddRightDistributivityFactoring( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `RightDistributivityFactoring`.  $F : (L, a) \mapsto \text{RightDistributivityFactoring}(L, a)$ .

#### 1.14.8 `AddRightDistributivityFactoringWithGivenObjects` (for `IsCapCategory`, `IsFunction`)

▷ `AddRightDistributivityFactoringWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `RightDistributivityFactoringWithGivenObjects`.  $F : (s, L, a, r) \mapsto \text{RightDistributivityFactoringWithGivenObjects}(s, L, a, r)$ .

#### 1.14.9 `AddBraiding` (for `IsCapCategory`, `IsFunction`)

▷ `AddBraiding(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `Braiding`.  $F : (a, b) \mapsto \text{Braiding}(a, b)$ .

#### 1.14.10 `AddBraidingInverse` (for `IsCapCategory`, `IsFunction`)

▷ `AddBraidingInverse(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `BraidingInverse`.  $F : (a, b) \mapsto \text{BraidingInverse}(a, b)$ .

#### 1.14.11 `AddBraidingInverseWithGivenTensorProducts` (for `IsCapCategory`, `IsFunction`)

▷ `AddBraidingInverseWithGivenTensorProducts(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `BraidingInverseWithGivenTensorProducts`.  $F : (s, a, b, r) \mapsto \text{BraidingInverseWithGivenTensorProducts}(s, a, b, r)$ .

#### 1.14.12 `AddBraidingWithGivenTensorProducts` (for `IsCapCategory`, `IsFunction`)

▷ `AddBraidingWithGivenTensorProducts(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `BraidingWithGivenTensorProducts`.  $F : (s, a, b, r) \mapsto \text{BraidingWithGivenTensorProducts}(s, a, b, r)$ .

### 1.14.13 AddClosedMonoidalLeftCoevaluationMorphism (for IsCapCategory, IsFunction)

▷ AddClosedMonoidalLeftCoevaluationMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation ClosedMonoidalLeftCoevaluationMorphism.  $F : (a, b) \mapsto \text{ClosedMonoidalLeftCoevaluationMorphism}(a, b)$ .

### 1.14.14 AddClosedMonoidalLeftCoevaluationMorphismWithGivenRange (for IsCapCategory, IsFunction)

▷ AddClosedMonoidalLeftCoevaluationMorphismWithGivenRange( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation ClosedMonoidalLeftCoevaluationMorphismWithGivenRange.  $F : (a, b, r) \mapsto \text{ClosedMonoidalLeftCoevaluationMorphismWithGivenRange}(a, b, r)$ .

### 1.14.15 AddClosedMonoidalLeftEvaluationMorphism (for IsCapCategory, IsFunction)

▷ AddClosedMonoidalLeftEvaluationMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation ClosedMonoidalLeftEvaluationMorphism.  $F : (a, b) \mapsto \text{ClosedMonoidalLeftEvaluationMorphism}(a, b)$ .

### 1.14.16 AddClosedMonoidalLeftEvaluationMorphismWithGivenSource (for IsCapCategory, IsFunction)

▷ AddClosedMonoidalLeftEvaluationMorphismWithGivenSource( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation ClosedMonoidalLeftEvaluationMorphismWithGivenSource.  $F : (a, b, s) \mapsto \text{ClosedMonoidalLeftEvaluationMorphismWithGivenSource}(a, b, s)$ .

### 1.14.17 AddClosedMonoidalRightCoevaluationMorphism (for IsCapCategory, IsFunction)

▷ AddClosedMonoidalRightCoevaluationMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation ClosedMonoidalRightCoevaluationMorphism.  $F : (a, b) \mapsto \text{ClosedMonoidalRightCoevaluationMorphism}(a, b)$ .



### 1.14.18 AddClosedMonoidalRightCoevaluationMorphismWithGivenRange (for IsCapCategory, IsFunction)

▷ AddClosedMonoidalRightCoevaluationMorphismWithGivenRange( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation ClosedMonoidalRightCoevaluationMorphismWithGivenRange.  $F : (a, b, r) \mapsto \text{ClosedMonoidalRightCoevaluationMorphismWithGivenRange}(a, b, r)$ .

### 1.14.19 AddClosedMonoidalRightEvaluationMorphism (for IsCapCategory, IsFunction)

▷ AddClosedMonoidalRightEvaluationMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation ClosedMonoidalRightEvaluationMorphism.  $F : (a, b) \mapsto \text{ClosedMonoidalRightEvaluationMorphism}(a, b)$ .

### 1.14.20 AddClosedMonoidalRightEvaluationMorphismWithGivenSource (for IsCapCategory, IsFunction)

▷ AddClosedMonoidalRightEvaluationMorphismWithGivenSource( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation ClosedMonoidalRightEvaluationMorphismWithGivenSource.  $F : (a, b, s) \mapsto \text{ClosedMonoidalRightEvaluationMorphismWithGivenSource}(a, b, s)$ .

### 1.14.21 AddDualOnMorphisms (for IsCapCategory, IsFunction)

▷ AddDualOnMorphisms( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation DualOnMorphisms.  $F : (\alpha) \mapsto \text{DualOnMorphisms}(\alpha)$ .

### 1.14.22 AddDualOnMorphismsWithGivenDuals (for IsCapCategory, IsFunction)

▷ AddDualOnMorphismsWithGivenDuals( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation DualOnMorphismsWithGivenDuals.  $F : (s, \alpha, r) \mapsto \text{DualOnMorphismsWithGivenDuals}(s, \alpha, r)$ .

### 1.14.23 AddDualOnObjects (for IsCapCategory, IsFunction)

▷ AddDualOnObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation DualOnObjects.  $F : (a) \mapsto \text{DualOnObjects}(a)$ .

**1.14.24 AddEvaluationForDual (for IsCapCategory, IsFunction)**

▷ AddEvaluationForDual( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation EvaluationForDual.  $F : (a) \mapsto \text{EvaluationForDual}(a)$ .

**1.14.25 AddEvaluationForDualWithGivenTensorProduct (for IsCapCategory, IsFunction)**

▷ AddEvaluationForDualWithGivenTensorProduct( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation EvaluationForDualWithGivenTensorProduct.  $F : (s, a, r) \mapsto \text{EvaluationForDualWithGivenTensorProduct}(s, a, r)$ .

**1.14.26 AddInternalHomOnMorphisms (for IsCapCategory, IsFunction)**

▷ AddInternalHomOnMorphisms( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalHomOnMorphisms.  $F : (\alpha, \beta) \mapsto \text{InternalHomOnMorphisms}(\alpha, \beta)$ .

**1.14.27 AddInternalHomOnMorphismsWithGivenInternalHoms (for IsCapCategory, IsFunction)**

▷ AddInternalHomOnMorphismsWithGivenInternalHoms( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalHomOnMorphismsWithGivenInternalHoms.  $F : (s, \alpha, \beta, r) \mapsto \text{InternalHomOnMorphismsWithGivenInternalHoms}(s, \alpha, \beta, r)$ .

**1.14.28 AddInternalHomOnObjects (for IsCapCategory, IsFunction)**

▷ AddInternalHomOnObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalHomOnObjects.  $F : (a, b) \mapsto \text{InternalHomOnObjects}(a, b)$ .

**1.14.29 AddInternalHomToTensorProductLeftAdjunctMorphism (for IsCapCategory, IsFunction)**

▷ AddInternalHomToTensorProductLeftAdjunctMorphism( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalHomToTensorProductLeftAdjunctMorphism`.  $F : (b, c, g) \mapsto \text{InternalHomToTensorProductLeftAdjunctMorphism}(b, c, g)$ .

#### 1.14.30 AddInternalHomToTensorProductLeftAdjunctMorphismWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ `AddInternalHomToTensorProductLeftAdjunctMorphismWithGivenTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalHomToTensorProductLeftAdjunctMorphismWithGivenTensorProduct`.  $F : (b, c, g, s) \mapsto \text{InternalHomToTensorProductLeftAdjunctMorphismWithGivenTensorProduct}(b, c, g, s)$ .

#### 1.14.31 AddInternalHomToTensorProductRightAdjunctMorphism (for IsCapCategory, IsFunction)

▷ `AddInternalHomToTensorProductRightAdjunctMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalHomToTensorProductRightAdjunctMorphism`.  $F : (a, c, g) \mapsto \text{InternalHomToTensorProductRightAdjunctMorphism}(a, c, g)$ .

#### 1.14.32 AddInternalHomToTensorProductRightAdjunctMorphismWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ `AddInternalHomToTensorProductRightAdjunctMorphismWithGivenTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalHomToTensorProductRightAdjunctMorphismWithGivenTensorProduct`.  $F : (a, c, g, s) \mapsto \text{InternalHomToTensorProductRightAdjunctMorphismWithGivenTensorProduct}(a, c, g, s)$ .

#### 1.14.33 AddIsomorphismFromDualObjectToInternalHomIntoTensorUnit (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromDualObjectToInternalHomIntoTensorUnit(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromDualObjectToInternalHomIntoTensorUnit`.  $F : (a) \mapsto \text{IsomorphismFromDualObjectToInternalHomIntoTensorUnit}(a)$ .

### 1.14.34 AddIsomorphismFromInternalHomIntoTensorUnitToDualObject (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromInternalHomIntoTensorUnitToDualObject( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromInternalHomIntoTensorUnitToDualObject.  $F : (a) \mapsto \text{IsomorphismFromInternalHomIntoTensorUnitToDualObject}(a)$ .

### 1.14.35 AddIsomorphismFromInternalHomToObject (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromInternalHomToObject( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromInternalHomToObject.  $F : (a) \mapsto \text{IsomorphismFromInternalHomToObject}(a)$ .

### 1.14.36 AddIsomorphismFromInternalHomToObjectWithGivenInternalHom (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromInternalHomToObjectWithGivenInternalHom( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromInternalHomToObjectWithGivenInternalHom.  $F : (a, s) \mapsto \text{IsomorphismFromInternalHomToObjectWithGivenInternalHom}(a, s)$ .

### 1.14.37 AddIsomorphismFromObjectToInternalHom (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromObjectToInternalHom( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromObjectToInternalHom.  $F : (a) \mapsto \text{IsomorphismFromObjectToInternalHom}(a)$ .

### 1.14.38 AddIsomorphismFromObjectToInternalHomWithGivenInternalHom (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromObjectToInternalHomWithGivenInternalHom( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromObjectToInternalHomWithGivenInternalHom.  $F : (a, r) \mapsto \text{IsomorphismFromObjectToInternalHomWithGivenInternalHom}(a, r)$ .

### 1.14.39 AddLambdaElimination (for IsCapCategory, IsFunction)

▷ AddLambdaElimination( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LambdaElimination.  $F : (a, b, \alpha) \mapsto \text{LambdaElimination}(a, b, \alpha)$ .

### 1.14.40 AddLambdaIntroduction (for IsCapCategory, IsFunction)

▷ AddLambdaIntroduction( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LambdaIntroduction.  $F : (\alpha) \mapsto \text{LambdaIntroduction}(\alpha)$ .

### 1.14.41 AddMonoidalPostComposeMorphism (for IsCapCategory, IsFunction)

▷ AddMonoidalPostComposeMorphism( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MonoidalPostComposeMorphism.  $F : (a, b, c) \mapsto \text{MonoidalPostComposeMorphism}(a, b, c)$ .

### 1.14.42 AddMonoidalPostComposeMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddMonoidalPostComposeMorphismWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MonoidalPostComposeMorphismWithGivenObjects.  $F : (s, a, b, c, r) \mapsto \text{MonoidalPostComposeMorphismWithGivenObjects}(s, a, b, c, r)$ .

### 1.14.43 AddMonoidalPreComposeMorphism (for IsCapCategory, IsFunction)

▷ AddMonoidalPreComposeMorphism( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MonoidalPreComposeMorphism.  $F : (a, b, c) \mapsto \text{MonoidalPreComposeMorphism}(a, b, c)$ .

### 1.14.44 AddMonoidalPreComposeMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddMonoidalPreComposeMorphismWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MonoidalPreComposeMorphismWithGivenObjects`.  $F : (s, a, b, c, r) \mapsto \text{MonoidalPreComposeMorphismWithGivenObjects}(s, a, b, c, r)$ .

#### 1.14.45 AddMorphismFromTensorProductToInternalHom (for IsCapCategory, IsFunction)

▷ `AddMorphismFromTensorProductToInternalHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromTensorProductToInternalHom`.  $F : (a, b) \mapsto \text{MorphismFromTensorProductToInternalHom}(a, b)$ .

#### 1.14.46 AddMorphismFromTensorProductToInternalHomWithGivenObjects (for IsCapCategory, IsFunction)

▷ `AddMorphismFromTensorProductToInternalHomWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromTensorProductToInternalHomWithGivenObjects`.  $F : (s, a, b, r) \mapsto \text{MorphismFromTensorProductToInternalHomWithGivenObjects}(s, a, b, r)$ .

#### 1.14.47 AddMorphismToBidual (for IsCapCategory, IsFunction)

▷ `AddMorphismToBidual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismToBidual`.  $F : (a) \mapsto \text{MorphismToBidual}(a)$ .

#### 1.14.48 AddMorphismToBidualWithGivenBidual (for IsCapCategory, IsFunction)

▷ `AddMorphismToBidualWithGivenBidual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismToBidualWithGivenBidual`.  $F : (a, r) \mapsto \text{MorphismToBidualWithGivenBidual}(a, r)$ .

#### 1.14.49 AddTensorProductDualityCompatibilityMorphism (for IsCapCategory, IsFunction)

▷ `AddTensorProductDualityCompatibilityMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductDualityCompatibilityMorphism`.  $F : (a, b) \mapsto \text{TensorProductDualityCompatibilityMorphism}(a, b)$ .

### 1.14.50 AddTensorProductDualityCompatibilityMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddTensorProductDualityCompatibilityMorphismWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductDualityCompatibilityMorphismWithGivenObjects.  $F : (s, a, b, r) \mapsto \text{TensorProductDualityCompatibilityMorphismWithGivenObjects}(s, a, b, r)$ .

### 1.14.51 AddTensorProductInternalHomCompatibilityMorphism (for IsCapCategory, IsFunction)

▷ AddTensorProductInternalHomCompatibilityMorphism( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductInternalHomCompatibilityMorphism.  $F : (list) \mapsto \text{TensorProductInternalHomCompatibilityMorphism}(list)$ .

### 1.14.52 AddTensorProductInternalHomCompatibilityMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddTensorProductInternalHomCompatibilityMorphismWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductInternalHomCompatibilityMorphismWithGivenObjects.  $F : (source, list, range) \mapsto \text{TensorProductInternalHomCompatibilityMorphismWithGivenObjects}(source, list, range)$ .

### 1.14.53 AddTensorProductToInternalHomLeftAdjunctMorphism (for IsCapCategory, IsFunction)

▷ AddTensorProductToInternalHomLeftAdjunctMorphism( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductToInternalHomLeftAdjunctMorphism.  $F : (a, b, f) \mapsto \text{TensorProductToInternalHomLeftAdjunctMorphism}(a, b, f)$ .

### 1.14.54 AddTensorProductToInternalHomLeftAdjunctMorphismWithGivenInternalHom (for IsCapCategory, IsFunction)

▷ AddTensorProductToInternalHomLeftAdjunctMorphismWithGivenInternalHom( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation

TensorProductToInternalHomLeftAdjunctMorphismWithGivenInternalHom.  $F : (a, b, f, i) \mapsto$   
 TensorProductToInternalHomLeftAdjunctMorphismWithGivenInternalHom( $a, b, f, i$ ).

#### 1.14.55 AddTensorProductToInternalHomRightAdjunctMorphism (for IsCapCategory, IsFunction)

▷ AddTensorProductToInternalHomRightAdjunctMorphism( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductToInternalHomRightAdjunctMorphism.  $F : (a, b, f) \mapsto$  TensorProductToInternalHomRightAdjunctMorphism( $a, b, f$ ).

#### 1.14.56 AddTensorProductToInternalHomRightAdjunctMorphismWithGivenInternalHom (for IsCapCategory, IsFunction)

▷ AddTensorProductToInternalHomRightAdjunctMorphismWithGivenInternalHom( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductToInternalHomRightAdjunctMorphismWithGivenInternalHom.  $F : (a, b, f, i) \mapsto$  TensorProductToInternalHomRightAdjunctMorphismWithGivenInternalHom( $a, b, f, i$ ).

#### 1.14.57 AddUniversalPropertyOfDual (for IsCapCategory, IsFunction)

▷ AddUniversalPropertyOfDual( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation UniversalPropertyOfDual.  $F : (t, a, alpha) \mapsto$  UniversalPropertyOfDual( $t, a, alpha$ ).

#### 1.14.58 AddCoDualOnMorphisms (for IsCapCategory, IsFunction)

▷ AddCoDualOnMorphisms( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoDualOnMorphisms.  $F : (alpha) \mapsto$  CoDualOnMorphisms( $alpha$ ).

#### 1.14.59 AddCoDualOnMorphismsWithGivenCoDuals (for IsCapCategory, IsFunction)

▷ AddCoDualOnMorphismsWithGivenCoDuals( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoDualOnMorphismsWithGivenCoDuals.  $F : (s, alpha, r) \mapsto$  CoDualOnMorphismsWithGivenCoDuals( $s, alpha, r$ ).



**1.14.60 AddCoDualOnObjects (for IsCapCategory, IsFunction)**

▷ AddCoDualOnObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoDualOnObjects.  $F : (a) \mapsto \text{CoDualOnObjects}(a)$ .

**1.14.61 AddCoDualityTensorProductCompatibilityMorphism (for IsCapCategory, IsFunction)**

▷ AddCoDualityTensorProductCompatibilityMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoDualityTensorProductCompatibilityMorphism.  $F : (a, b) \mapsto \text{CoDualityTensorProductCompatibilityMorphism}(a, b)$ .

**1.14.62 AddCoDualityTensorProductCompatibilityMorphismWithGivenObjects (for IsCapCategory, IsFunction)**

▷ AddCoDualityTensorProductCompatibilityMorphismWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoDualityTensorProductCompatibilityMorphismWithGivenObjects.  $F : (s, a, b, r) \mapsto \text{CoDualityTensorProductCompatibilityMorphismWithGivenObjects}(s, a, b, r)$ .

**1.14.63 AddCoLambdaElimination (for IsCapCategory, IsFunction)**

▷ AddCoLambdaElimination( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoLambdaElimination.  $F : (a, b, \alpha) \mapsto \text{CoLambdaElimination}(a, b, \alpha)$ .

**1.14.64 AddCoLambdaIntroduction (for IsCapCategory, IsFunction)**

▷ AddCoLambdaIntroduction( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoLambdaIntroduction.  $F : (\alpha) \mapsto \text{CoLambdaIntroduction}(\alpha)$ .

**1.14.65 AddCoclosedEvaluationForCoDual (for IsCapCategory, IsFunction)**

▷ AddCoclosedEvaluationForCoDual( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoclosedEvaluationForCoDual`.  $F : (a) \mapsto \text{CoclosedEvaluationForCoDual}(a)$ .

#### 1.14.66 AddCoclosedEvaluationForCoDualWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ `AddCoclosedEvaluationForCoDualWithGivenTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoclosedEvaluationForCoDualWithGivenTensorProduct`.  $F : (s, a, r) \mapsto \text{CoclosedEvaluationForCoDualWithGivenTensorProduct}(s, a, r)$ .

#### 1.14.67 AddCoclosedMonoidalLeftCoevaluationMorphism (for IsCapCategory, IsFunction)

▷ `AddCoclosedMonoidalLeftCoevaluationMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoclosedMonoidalLeftCoevaluationMorphism`.  $F : (a, b) \mapsto \text{CoclosedMonoidalLeftCoevaluationMorphism}(a, b)$ .

#### 1.14.68 AddCoclosedMonoidalLeftCoevaluationMorphismWithGivenSource (for IsCapCategory, IsFunction)

▷ `AddCoclosedMonoidalLeftCoevaluationMorphismWithGivenSource(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoclosedMonoidalLeftCoevaluationMorphismWithGivenSource`.  $F : (a, b, s) \mapsto \text{CoclosedMonoidalLeftCoevaluationMorphismWithGivenSource}(a, b, s)$ .

#### 1.14.69 AddCoclosedMonoidalLeftEvaluationMorphism (for IsCapCategory, IsFunction)

▷ `AddCoclosedMonoidalLeftEvaluationMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoclosedMonoidalLeftEvaluationMorphism`.  $F : (a, b) \mapsto \text{CoclosedMonoidalLeftEvaluationMorphism}(a, b)$ .

#### 1.14.70 AddCoclosedMonoidalLeftEvaluationMorphismWithGivenRange (for IsCapCategory, IsFunction)

▷ `AddCoclosedMonoidalLeftEvaluationMorphismWithGivenRange(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoclosedMonoidalLeftEvaluationMorphismWithGivenRange`.  $F : (a, b, r) \mapsto \text{CoclosedMonoidalLeftEvaluationMorphismWithGivenRange}(a, b, r)$ .

#### 1.14.71 AddCoclosedMonoidalRightCoevaluationMorphism (for IsCapCategory, IsFunction)

▷ `AddCoclosedMonoidalRightCoevaluationMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoclosedMonoidalRightCoevaluationMorphism`.  $F : (a, b) \mapsto \text{CoclosedMonoidalRightCoevaluationMorphism}(a, b)$ .

#### 1.14.72 AddCoclosedMonoidalRightCoevaluationMorphismWithGivenSource (for IsCapCategory, IsFunction)

▷ `AddCoclosedMonoidalRightCoevaluationMorphismWithGivenSource(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoclosedMonoidalRightCoevaluationMorphismWithGivenSource`.  $F : (a, b, s) \mapsto \text{CoclosedMonoidalRightCoevaluationMorphismWithGivenSource}(a, b, s)$ .

#### 1.14.73 AddCoclosedMonoidalRightEvaluationMorphism (for IsCapCategory, IsFunction)

▷ `AddCoclosedMonoidalRightEvaluationMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoclosedMonoidalRightEvaluationMorphism`.  $F : (a, b) \mapsto \text{CoclosedMonoidalRightEvaluationMorphism}(a, b)$ .

#### 1.14.74 AddCoclosedMonoidalRightEvaluationMorphismWithGivenRange (for IsCapCategory, IsFunction)

▷ `AddCoclosedMonoidalRightEvaluationMorphismWithGivenRange(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoclosedMonoidalRightEvaluationMorphismWithGivenRange`.  $F : (a, b, r) \mapsto \text{CoclosedMonoidalRightEvaluationMorphismWithGivenRange}(a, b, r)$ .

#### 1.14.75 AddInternalCoHomOnMorphisms (for IsCapCategory, IsFunction)

▷ `AddInternalCoHomOnMorphisms(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalCoHomOnMorphisms`.  $F : (\alpha, \beta) \mapsto \text{InternalCoHomOnMorphisms}(\alpha, \beta)$ .

### 1.14.76 AddInternalCoHomOnMorphismsWithGivenInternalCoHoms (for IsCapCategory, IsFunction)

▷ AddInternalCoHomOnMorphismsWithGivenInternalCoHoms( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalCoHomOnMorphismsWithGivenInternalCoHoms.  $F : (s, \alpha, \beta, r) \mapsto \text{InternalCoHomOnMorphismsWithGivenInternalCoHoms}(s, \alpha, \beta, r)$ .

### 1.14.77 AddInternalCoHomOnObjects (for IsCapCategory, IsFunction)

▷ AddInternalCoHomOnObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalCoHomOnObjects.  $F : (a, b) \mapsto \text{InternalCoHomOnObjects}(a, b)$ .

### 1.14.78 AddInternalCoHomTensorProductCompatibilityMorphism (for IsCapCategory, IsFunction)

▷ AddInternalCoHomTensorProductCompatibilityMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalCoHomTensorProductCompatibilityMorphism.  $F : (list) \mapsto \text{InternalCoHomTensorProductCompatibilityMorphism}(list)$ .

### 1.14.79 AddInternalCoHomTensorProductCompatibilityMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddInternalCoHomTensorProductCompatibilityMorphismWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalCoHomTensorProductCompatibilityMorphismWithGivenObjects.  $F : (source, list, range) \mapsto \text{InternalCoHomTensorProductCompatibilityMorphismWithGivenObjects}(source, list, range)$ .

### 1.14.80 AddInternalCoHomToTensorProductLeftAdjunctMorphism (for IsCapCategory, IsFunction)

▷ AddInternalCoHomToTensorProductLeftAdjunctMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalCoHomToTensorProductLeftAdjunctMorphism.  $F : (a, c, f) \mapsto \text{InternalCoHomToTensorProductLeftAdjunctMorphism}(a, c, f)$ .

### 1.14.81 AddInternalCoHomToTensorProductLeftAdjunctMorphismWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ AddInternalCoHomToTensorProductLeftAdjunctMorphismWithGivenTensorProduct( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalCoHomToTensorProductLeftAdjunctMorphismWithGivenTensorProduct.  $F : (a, c, f, t) \mapsto \text{InternalCoHomToTensorProductLeftAdjunctMorphismWithGivenTensorProduct}(a, c, f, t)$ .

### 1.14.82 AddInternalCoHomToTensorProductRightAdjunctMorphism (for IsCapCategory, IsFunction)

▷ AddInternalCoHomToTensorProductRightAdjunctMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalCoHomToTensorProductRightAdjunctMorphism.  $F : (a, b, f) \mapsto \text{InternalCoHomToTensorProductRightAdjunctMorphism}(a, b, f)$ .

### 1.14.83 AddInternalCoHomToTensorProductRightAdjunctMorphismWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ AddInternalCoHomToTensorProductRightAdjunctMorphismWithGivenTensorProduct( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalCoHomToTensorProductRightAdjunctMorphismWithGivenTensorProduct.  $F : (a, b, f, t) \mapsto \text{InternalCoHomToTensorProductRightAdjunctMorphismWithGivenTensorProduct}(a, b, f, t)$ .

### 1.14.84 AddIsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit.  $F : (a) \mapsto \text{IsomorphismFromCoDualObjectToInternalCoHomFromTensorUnit}(a)$ .

### 1.14.85 AddIsomorphismFromInternalCoHomFromTensorUnitToCoDualObject (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromInternalCoHomFromTensorUnitToCoDualObject( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromInternalCoHomFromTensorUnitToCoDualObject`.  $F : (a) \mapsto \text{IsomorphismFromInternalCoHomFromTensorUnitToCoDualObject}(a)$ .

#### 1.14.86 AddIsomorphismFromInternalCoHomToObject (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromInternalCoHomToObject(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromInternalCoHomToObject`.  $F : (a) \mapsto \text{IsomorphismFromInternalCoHomToObject}(a)$ .

#### 1.14.87 AddIsomorphismFromInternalCoHomToObjectWithGivenInternalCoHom (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromInternalCoHomToObjectWithGivenInternalCoHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromInternalCoHomToObjectWithGivenInternalCoHom`.  $F : (a, s) \mapsto \text{IsomorphismFromInternalCoHomToObjectWithGivenInternalCoHom}(a, s)$ .

#### 1.14.88 AddIsomorphismFromObjectToInternalCoHom (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromObjectToInternalCoHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromObjectToInternalCoHom`.  $F : (a) \mapsto \text{IsomorphismFromObjectToInternalCoHom}(a)$ .

#### 1.14.89 AddIsomorphismFromObjectToInternalCoHomWithGivenInternalCoHom (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromObjectToInternalCoHomWithGivenInternalCoHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromObjectToInternalCoHomWithGivenInternalCoHom`.  $F : (a, r) \mapsto \text{IsomorphismFromObjectToInternalCoHomWithGivenInternalCoHom}(a, r)$ .

#### 1.14.90 AddMonoidalPostCoComposeMorphism (for IsCapCategory, IsFunction)

▷ `AddMonoidalPostCoComposeMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MonoidalPostCoComposeMorphism`.  $F : (a, b, c) \mapsto \text{MonoidalPostCoComposeMorphism}(a, b, c)$ .

#### 1.14.91 AddMonoidalPostCoComposeMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ `AddMonoidalPostCoComposeMorphismWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MonoidalPostCoComposeMorphismWithGivenObjects`.  $F : (s, a, b, c, r) \mapsto \text{MonoidalPostCoComposeMorphismWithGivenObjects}(s, a, b, c, r)$ .

#### 1.14.92 AddMonoidalPreCoComposeMorphism (for IsCapCategory, IsFunction)

▷ `AddMonoidalPreCoComposeMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MonoidalPreCoComposeMorphism`.  $F : (a, b, c) \mapsto \text{MonoidalPreCoComposeMorphism}(a, b, c)$ .

#### 1.14.93 AddMonoidalPreCoComposeMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ `AddMonoidalPreCoComposeMorphismWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MonoidalPreCoComposeMorphismWithGivenObjects`.  $F : (s, a, b, c, r) \mapsto \text{MonoidalPreCoComposeMorphismWithGivenObjects}(s, a, b, c, r)$ .

#### 1.14.94 AddMorphismFromCoBidual (for IsCapCategory, IsFunction)

▷ `AddMorphismFromCoBidual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromCoBidual`.  $F : (a) \mapsto \text{MorphismFromCoBidual}(a)$ .

#### 1.14.95 AddMorphismFromCoBidualWithGivenCoBidual (for IsCapCategory, IsFunction)

▷ `AddMorphismFromCoBidualWithGivenCoBidual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromCoBidualWithGivenCoBidual`.  $F : (a, s) \mapsto \text{MorphismFromCoBidualWithGivenCoBidual}(a, s)$ .

#### 1.14.96 AddMorphismFromInternalCoHomToTensorProduct (for IsCapCategory, IsFunction)

▷ AddMorphismFromInternalCoHomToTensorProduct( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromInternalCoHomToTensorProduct.  $F : (a, b) \mapsto \text{MorphismFromInternalCoHomToTensorProduct}(a, b)$ .

#### 1.14.97 AddMorphismFromInternalCoHomToTensorProductWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddMorphismFromInternalCoHomToTensorProductWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromInternalCoHomToTensorProductWithGivenObjects.  $F : (s, a, b, r) \mapsto \text{MorphismFromInternalCoHomToTensorProductWithGivenObjects}(s, a, b, r)$ .

#### 1.14.98 AddTensorProductToInternalCoHomLeftAdjunctMorphism (for IsCapCategory, IsFunction)

▷ AddTensorProductToInternalCoHomLeftAdjunctMorphism( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductToInternalCoHomLeftAdjunctMorphism.  $F : (b, c, g) \mapsto \text{TensorProductToInternalCoHomLeftAdjunctMorphism}(b, c, g)$ .

#### 1.14.99 AddTensorProductToInternalCoHomLeftAdjunctMorphismWithGivenInternalCoHom (for IsCapCategory, IsFunction)

▷ AddTensorProductToInternalCoHomLeftAdjunctMorphismWithGivenInternalCoHom( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductToInternalCoHomLeftAdjunctMorphismWithGivenInternalCoHom.  $F : (b, c, g, i) \mapsto \text{TensorProductToInternalCoHomLeftAdjunctMorphismWithGivenInternalCoHom}(b, c, g, i)$ .

#### 1.14.100 AddTensorProductToInternalCoHomRightAdjunctMorphism (for IsCapCategory, IsFunction)

▷ AddTensorProductToInternalCoHomRightAdjunctMorphism( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductToInternalCoHomRightAdjunctMorphism.  $F : (b, c, g) \mapsto \text{TensorProductToInternalCoHomRightAdjunctMorphism}(b, c, g)$ .



### 1.14.101 AddTensorProductToInternalCoHomRightAdjunctMorphismWithGivenInternalCoHom (for IsCapCategory, IsFunction)

▷ AddTensorProductToInternalCoHomRightAdjunctMorphismWithGivenInternalCoHom( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductToInternalCoHomRightAdjunctMorphismWithGivenInternalCoHom.  $F : (b, c, g, i) \mapsto \text{TensorProductToInternalCoHomRightAdjunctMorphismWithGivenInternalCoHom}(b, c, g, i)$ .

### 1.14.102 AddUniversalPropertyOfCoDual (for IsCapCategory, IsFunction)

▷ AddUniversalPropertyOfCoDual( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation UniversalPropertyOfCoDual.  $F : (t, a, \alpha) \mapsto \text{UniversalPropertyOfCoDual}(t, a, \alpha)$ .

### 1.14.103 AddIsomorphismFromLeftDualObjectToLeftInternalHomIntoTensorUnit (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromLeftDualObjectToLeftInternalHomIntoTensorUnit( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromLeftDualObjectToLeftInternalHomIntoTensorUnit.  $F : (a) \mapsto \text{IsomorphismFromLeftDualObjectToLeftInternalHomIntoTensorUnit}(a)$ .

### 1.14.104 AddIsomorphismFromLeftInternalHomIntoTensorUnitToLeftDualObject (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromLeftInternalHomIntoTensorUnitToLeftDualObject( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromLeftInternalHomIntoTensorUnitToLeftDualObject.  $F : (a) \mapsto \text{IsomorphismFromLeftInternalHomIntoTensorUnitToLeftDualObject}(a)$ .

### 1.14.105 AddIsomorphismFromLeftInternalHomToObject (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromLeftInternalHomToObject( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromLeftInternalHomToObject.  $F : (a) \mapsto \text{IsomorphismFromLeftInternalHomToObject}(a)$ .

#### 1.14.106 AddIsomorphismFromLeftInternalHomToObjectWithGivenLeftInternalHom (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromLeftInternalHomToObjectWithGivenLeftInternalHom( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromLeftInternalHomToObjectWithGivenLeftInternalHom.  $F : (a, s) \mapsto \text{IsomorphismFromLeftInternalHomToObjectWithGivenLeftInternalHom}(a, s)$ .

#### 1.14.107 AddIsomorphismFromObjectToLeftInternalHom (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromObjectToLeftInternalHom( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromObjectToLeftInternalHom.  $F : (a) \mapsto \text{IsomorphismFromObjectToLeftInternalHom}(a)$ .

#### 1.14.108 AddIsomorphismFromObjectToLeftInternalHomWithGivenLeftInternalHom (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromObjectToLeftInternalHomWithGivenLeftInternalHom( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromObjectToLeftInternalHomWithGivenLeftInternalHom.  $F : (a, r) \mapsto \text{IsomorphismFromObjectToLeftInternalHomWithGivenLeftInternalHom}(a, r)$ .

#### 1.14.109 AddLeftClosedMonoidalCoevaluationMorphism (for IsCapCategory, IsFunction)

▷ AddLeftClosedMonoidalCoevaluationMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftClosedMonoidalCoevaluationMorphism.  $F : (a, b) \mapsto \text{LeftClosedMonoidalCoevaluationMorphism}(a, b)$ .

#### 1.14.110 AddLeftClosedMonoidalCoevaluationMorphismWithGivenRange (for IsCapCategory, IsFunction)

▷ AddLeftClosedMonoidalCoevaluationMorphismWithGivenRange( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftClosedMonoidalCoevaluationMorphismWithGivenRange.  $F : (a, b, r) \mapsto \text{LeftClosedMonoidalCoevaluationMorphismWithGivenRange}(a, b, r)$ .

#### 1.14.111 AddLeftClosedMonoidalEvaluationForLeftDual (for IsCapCategory, IsFunction)

▷ AddLeftClosedMonoidalEvaluationForLeftDual( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftClosedMonoidalEvaluationForLeftDual.  $F : (a) \mapsto \text{LeftClosedMonoidalEvaluationForLeftDual}(a)$ .

#### 1.14.112 AddLeftClosedMonoidalEvaluationForLeftDualWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ AddLeftClosedMonoidalEvaluationForLeftDualWithGivenTensorProduct( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftClosedMonoidalEvaluationForLeftDualWithGivenTensorProduct.  $F : (s, a, r) \mapsto \text{LeftClosedMonoidalEvaluationForLeftDualWithGivenTensorProduct}(s, a, r)$ .

#### 1.14.113 AddLeftClosedMonoidalEvaluationMorphism (for IsCapCategory, IsFunction)

▷ AddLeftClosedMonoidalEvaluationMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftClosedMonoidalEvaluationMorphism.  $F : (a, b) \mapsto \text{LeftClosedMonoidalEvaluationMorphism}(a, b)$ .

#### 1.14.114 AddLeftClosedMonoidalEvaluationMorphismWithGivenSource (for IsCapCategory, IsFunction)

▷ AddLeftClosedMonoidalEvaluationMorphismWithGivenSource( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftClosedMonoidalEvaluationMorphismWithGivenSource.  $F : (a, b, s) \mapsto \text{LeftClosedMonoidalEvaluationMorphismWithGivenSource}(a, b, s)$ .

#### 1.14.115 AddLeftClosedMonoidalLambdaElimination (for IsCapCategory, IsFunction)

▷ AddLeftClosedMonoidalLambdaElimination( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftClosedMonoidalLambdaElimination.  $F : (a, b, \alpha) \mapsto \text{LeftClosedMonoidalLambdaElimination}(a, b, \alpha)$ .

#### 1.14.116 AddLeftClosedMonoidalLambdaIntroduction (for IsCapCategory, IsFunction)

▷ AddLeftClosedMonoidalLambdaIntroduction( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftClosedMonoidalLambdaIntroduction.  $F : (\alpha) \mapsto \text{LeftClosedMonoidalLambdaIntroduction}(\alpha)$ .

#### 1.14.117 AddLeftClosedMonoidalPostComposeMorphism (for IsCapCategory, IsFunction)

▷ AddLeftClosedMonoidalPostComposeMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftClosedMonoidalPostComposeMorphism.  $F : (a, b, c) \mapsto \text{LeftClosedMonoidalPostComposeMorphism}(a, b, c)$ .

#### 1.14.118 AddLeftClosedMonoidalPostComposeMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddLeftClosedMonoidalPostComposeMorphismWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftClosedMonoidalPostComposeMorphismWithGivenObjects.  $F : (s, a, b, c, r) \mapsto \text{LeftClosedMonoidalPostComposeMorphismWithGivenObjects}(s, a, b, c, r)$ .

#### 1.14.119 AddLeftClosedMonoidalPreComposeMorphism (for IsCapCategory, IsFunction)

▷ AddLeftClosedMonoidalPreComposeMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftClosedMonoidalPreComposeMorphism.  $F : (a, b, c) \mapsto \text{LeftClosedMonoidalPreComposeMorphism}(a, b, c)$ .

#### 1.14.120 AddLeftClosedMonoidalPreComposeMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddLeftClosedMonoidalPreComposeMorphismWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftClosedMonoidalPreComposeMorphismWithGivenObjects.  $F : (s, a, b, c, r) \mapsto \text{LeftClosedMonoidalPreComposeMorphismWithGivenObjects}(s, a, b, c, r)$ .

**1.14.121 AddLeftDualOnMorphisms (for IsCapCategory, IsFunction)**

▷ AddLeftDualOnMorphisms( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftDualOnMorphisms.  $F : (alpha) \mapsto \text{LeftDualOnMorphisms}(alpha)$ .

**1.14.122 AddLeftDualOnMorphismsWithGivenLeftDuals (for IsCapCategory, IsFunction)**

▷ AddLeftDualOnMorphismsWithGivenLeftDuals( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftDualOnMorphismsWithGivenLeftDuals.  $F : (s, alpha, r) \mapsto \text{LeftDualOnMorphismsWithGivenLeftDuals}(s, alpha, r)$ .

**1.14.123 AddLeftDualOnObjects (for IsCapCategory, IsFunction)**

▷ AddLeftDualOnObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftDualOnObjects.  $F : (a) \mapsto \text{LeftDualOnObjects}(a)$ .

**1.14.124 AddLeftInternalHomOnMorphisms (for IsCapCategory, IsFunction)**

▷ AddLeftInternalHomOnMorphisms( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftInternalHomOnMorphisms.  $F : (alpha, beta) \mapsto \text{LeftInternalHomOnMorphisms}(alpha, beta)$ .

**1.14.125 AddLeftInternalHomOnMorphismsWithGivenLeftInternalHoms (for IsCapCategory, IsFunction)**

▷ AddLeftInternalHomOnMorphismsWithGivenLeftInternalHoms( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftInternalHomOnMorphismsWithGivenLeftInternalHoms.  $F : (s, alpha, beta, r) \mapsto \text{LeftInternalHomOnMorphismsWithGivenLeftInternalHoms}(s, alpha, beta, r)$ .

**1.14.126 AddLeftInternalHomOnObjects (for IsCapCategory, IsFunction)**

▷ AddLeftInternalHomOnObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `LeftInternalHomOnObjects`.  $F : (a,b) \mapsto \text{LeftInternalHomOnObjects}(a,b)$ .

#### 1.14.127 **AddLeftInternalHomToTensorProductAdjunctMorphism (for IsCapCategory, IsFunction)**

▷ `AddLeftInternalHomToTensorProductAdjunctMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `LeftInternalHomToTensorProductAdjunctMorphism`.  $F : (b,c,g) \mapsto \text{LeftInternalHomToTensorProductAdjunctMorphism}(b,c,g)$ .

#### 1.14.128 **AddLeftInternalHomToTensorProductAdjunctMorphismWithGivenTensorProduct (for IsCapCategory, IsFunction)**

▷ `AddLeftInternalHomToTensorProductAdjunctMorphismWithGivenTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `LeftInternalHomToTensorProductAdjunctMorphismWithGivenTensorProduct`.  $F : (b,c,g,t) \mapsto \text{LeftInternalHomToTensorProductAdjunctMorphismWithGivenTensorProduct}(b,c,g,t)$ .

#### 1.14.129 **AddMorphismFromTensorProductToLeftInternalHom (for IsCapCategory, IsFunction)**

▷ `AddMorphismFromTensorProductToLeftInternalHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromTensorProductToLeftInternalHom`.  $F : (a,b) \mapsto \text{MorphismFromTensorProductToLeftInternalHom}(a,b)$ .

#### 1.14.130 **AddMorphismFromTensorProductToLeftInternalHomWithGivenObjects (for IsCapCategory, IsFunction)**

▷ `AddMorphismFromTensorProductToLeftInternalHomWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromTensorProductToLeftInternalHomWithGivenObjects`.  $F : (s,a,b,r) \mapsto \text{MorphismFromTensorProductToLeftInternalHomWithGivenObjects}(s,a,b,r)$ .

#### 1.14.131 **AddMorphismToLeftBidual (for IsCapCategory, IsFunction)**

▷ `AddMorphismToLeftBidual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismToLeftBidual`.  $F : (a) \mapsto \text{MorphismToLeftBidual}(a)$ .

#### 1.14.132 **AddMorphismToLeftBidualWithGivenLeftBidual** (for **IsCapCategory**, **IsFunction**)

▷ `AddMorphismToLeftBidualWithGivenLeftBidual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismToLeftBidualWithGivenLeftBidual`.  $F : (a, r) \mapsto \text{MorphismToLeftBidualWithGivenLeftBidual}(a, r)$ .

#### 1.14.133 **AddTensorProductLeftDualityCompatibilityMorphism** (for **IsCapCategory**, **IsFunction**)

▷ `AddTensorProductLeftDualityCompatibilityMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductLeftDualityCompatibilityMorphism`.  $F : (a, b) \mapsto \text{TensorProductLeftDualityCompatibilityMorphism}(a, b)$ .

#### 1.14.134 **AddTensorProductLeftDualityCompatibilityMorphismWithGivenObjects** (for **IsCapCategory**, **IsFunction**)

▷ `AddTensorProductLeftDualityCompatibilityMorphismWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductLeftDualityCompatibilityMorphismWithGivenObjects`.  $F : (s, a, b, r) \mapsto \text{TensorProductLeftDualityCompatibilityMorphismWithGivenObjects}(s, a, b, r)$ .

#### 1.14.135 **AddTensorProductLeftInternalHomCompatibilityMorphism** (for **IsCapCategory**, **IsFunction**)

▷ `AddTensorProductLeftInternalHomCompatibilityMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductLeftInternalHomCompatibilityMorphism`.  $F : (list) \mapsto \text{TensorProductLeftInternalHomCompatibilityMorphism}(list)$ .

#### 1.14.136 **AddTensorProductLeftInternalHomCompatibilityMorphismWithGivenObjects** (for **IsCapCategory**, **IsFunction**)

▷ `AddTensorProductLeftInternalHomCompatibilityMorphismWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductLeftInternalHomCompatibilityMorphismWithGivenObjects`.  $F : (source, list, range) \mapsto \text{TensorProductLeftInternalHomCompatibilityMorphismWithGivenObjects}(source, list, range)$ .

#### 1.14.137 AddTensorProductToLeftInternalHomAdjunctMorphism (for IsCapCategory, IsFunction)

▷ `AddTensorProductToLeftInternalHomAdjunctMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductToLeftInternalHomAdjunctMorphism`.  $F : (a, b, f) \mapsto \text{TensorProductToLeftInternalHomAdjunctMorphism}(a, b, f)$ .

#### 1.14.138 AddTensorProductToLeftInternalHomAdjunctMorphismWithGivenLeftInternalHom (for IsCapCategory, IsFunction)

▷ `AddTensorProductToLeftInternalHomAdjunctMorphismWithGivenLeftInternalHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductToLeftInternalHomAdjunctMorphismWithGivenLeftInternalHom`.  $F : (a, b, f, i) \mapsto \text{TensorProductToLeftInternalHomAdjunctMorphismWithGivenLeftInternalHom}(a, b, f, i)$ .

#### 1.14.139 AddUniversalPropertyOfLeftDual (for IsCapCategory, IsFunction)

▷ `AddUniversalPropertyOfLeftDual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `UniversalPropertyOfLeftDual`.  $F : (t, a, alpha) \mapsto \text{UniversalPropertyOfLeftDual}(t, a, alpha)$ .

#### 1.14.140 AddIsomorphismFromLeftCoDualObjectToLeftInternalCoHomFromTensorUnit (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromLeftCoDualObjectToLeftInternalCoHomFromTensorUnit(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromLeftCoDualObjectToLeftInternalCoHomFromTensorUnit`.  $F : (a) \mapsto \text{IsomorphismFromLeftCoDualObjectToLeftInternalCoHomFromTensorUnit}(a)$ .



#### 1.14.141 AddIsomorphismFromLeftInternalCoHomFromTensorUnitToLeftCoDualObject (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromLeftInternalCoHomFromTensorUnitToLeftCoDualObject( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromLeftInternalCoHomFromTensorUnitToLeftCoDualObject.  $F : (a) \mapsto \text{IsomorphismFromLeftInternalCoHomFromTensorUnitToLeftCoDualObject}(a)$ .

#### 1.14.142 AddIsomorphismFromLeftInternalCoHomToObject (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromLeftInternalCoHomToObject( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromLeftInternalCoHomToObject.  $F : (a) \mapsto \text{IsomorphismFromLeftInternalCoHomToObject}(a)$ .

#### 1.14.143 AddIsomorphismFromLeftInternalCoHomToObjectWithGivenLeftInternalCoHom (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromLeftInternalCoHomToObjectWithGivenLeftInternalCoHom( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromLeftInternalCoHomToObjectWithGivenLeftInternalCoHom.  $F : (a, s) \mapsto \text{IsomorphismFromLeftInternalCoHomToObjectWithGivenLeftInternalCoHom}(a, s)$ .

#### 1.14.144 AddIsomorphismFromObjectToLeftInternalCoHom (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromObjectToLeftInternalCoHom( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation IsomorphismFromObjectToLeftInternalCoHom.  $F : (a) \mapsto \text{IsomorphismFromObjectToLeftInternalCoHom}(a)$ .

#### 1.14.145 AddIsomorphismFromObjectToLeftInternalCoHomWithGivenLeftInternalCoHom (for IsCapCategory, IsFunction)

▷ AddIsomorphismFromObjectToLeftInternalCoHomWithGivenLeftInternalCoHom( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation

IsomorphismFromObjectToLeftInternalCoHomWithGivenLeftInternalCoHom.  $F : (a, r) \mapsto \text{IsomorphismFromObjectToLeftInternalCoHomWithGivenLeftInternalCoHom}(a, r)$ .

#### 1.14.146 AddLeftCoDualOnMorphisms (for IsCapCategory, IsFunction)

▷ AddLeftCoDualOnMorphisms( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoDualOnMorphisms.  $F : (alpha) \mapsto \text{LeftCoDualOnMorphisms}(alpha)$ .

#### 1.14.147 AddLeftCoDualOnMorphismsWithGivenLeftCoDuals (for IsCapCategory, IsFunction)

▷ AddLeftCoDualOnMorphismsWithGivenLeftCoDuals( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoDualOnMorphismsWithGivenLeftCoDuals.  $F : (s, alpha, r) \mapsto \text{LeftCoDualOnMorphismsWithGivenLeftCoDuals}(s, alpha, r)$ .

#### 1.14.148 AddLeftCoDualOnObjects (for IsCapCategory, IsFunction)

▷ AddLeftCoDualOnObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoDualOnObjects.  $F : (a) \mapsto \text{LeftCoDualOnObjects}(a)$ .

#### 1.14.149 AddLeftCoDualityTensorProductCompatibilityMorphism (for IsCapCategory, IsFunction)

▷ AddLeftCoDualityTensorProductCompatibilityMorphism( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoDualityTensorProductCompatibilityMorphism.  $F : (a, b) \mapsto \text{LeftCoDualityTensorProductCompatibilityMorphism}(a, b)$ .

#### 1.14.150 AddLeftCoDualityTensorProductCompatibilityMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddLeftCoDualityTensorProductCompatibilityMorphismWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoDualityTensorProductCompatibilityMorphismWithGivenObjects.  $F : (s, a, b, r) \mapsto \text{LeftCoDualityTensorProductCompatibilityMorphismWithGivenObjects}(s, a, b, r)$ .

#### 1.14.151 AddLeftCoclosedMonoidalCoevaluationMorphism (for IsCapCategory, IsFunction)

▷ AddLeftCoclosedMonoidalCoevaluationMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoclosedMonoidalCoevaluationMorphism.  $F : (a, b) \mapsto \text{LeftCoclosedMonoidalCoevaluationMorphism}(a, b)$ .

#### 1.14.152 AddLeftCoclosedMonoidalCoevaluationMorphismWithGivenSource (for IsCapCategory, IsFunction)

▷ AddLeftCoclosedMonoidalCoevaluationMorphismWithGivenSource( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoclosedMonoidalCoevaluationMorphismWithGivenSource.  $F : (a, b, s) \mapsto \text{LeftCoclosedMonoidalCoevaluationMorphismWithGivenSource}(a, b, s)$ .

#### 1.14.153 AddLeftCoclosedMonoidalEvaluationForLeftCoDual (for IsCapCategory, IsFunction)

▷ AddLeftCoclosedMonoidalEvaluationForLeftCoDual( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoclosedMonoidalEvaluationForLeftCoDual.  $F : (a) \mapsto \text{LeftCoclosedMonoidalEvaluationForLeftCoDual}(a)$ .

#### 1.14.154 AddLeftCoclosedMonoidalEvaluationForLeftCoDualWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ AddLeftCoclosedMonoidalEvaluationForLeftCoDualWithGivenTensorProduct( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoclosedMonoidalEvaluationForLeftCoDualWithGivenTensorProduct.  $F : (s, a, r) \mapsto \text{LeftCoclosedMonoidalEvaluationForLeftCoDualWithGivenTensorProduct}(s, a, r)$ .

#### 1.14.155 AddLeftCoclosedMonoidalEvaluationMorphism (for IsCapCategory, IsFunction)

▷ AddLeftCoclosedMonoidalEvaluationMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoclosedMonoidalEvaluationMorphism.  $F : (a, b) \mapsto \text{LeftCoclosedMonoidalEvaluationMorphism}(a, b)$ .

### 1.14.156 AddLeftCoclosedMonoidalEvaluationMorphismWithGivenRange (for IsCapCategory, IsFunction)

▷ AddLeftCoclosedMonoidalEvaluationMorphismWithGivenRange( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoclosedMonoidalEvaluationMorphismWithGivenRange.  $F : (a, b, r) \mapsto \text{LeftCoclosedMonoidalEvaluationMorphismWithGivenRange}(a, b, r)$ .

### 1.14.157 AddLeftCoclosedMonoidalLambdaElimination (for IsCapCategory, IsFunction)

▷ AddLeftCoclosedMonoidalLambdaElimination( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoclosedMonoidalLambdaElimination.  $F : (a, b, \alpha) \mapsto \text{LeftCoclosedMonoidalLambdaElimination}(a, b, \alpha)$ .

### 1.14.158 AddLeftCoclosedMonoidalLambdaIntroduction (for IsCapCategory, IsFunction)

▷ AddLeftCoclosedMonoidalLambdaIntroduction( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoclosedMonoidalLambdaIntroduction.  $F : (\alpha) \mapsto \text{LeftCoclosedMonoidalLambdaIntroduction}(\alpha)$ .

### 1.14.159 AddLeftCoclosedMonoidalPostCoComposeMorphism (for IsCapCategory, IsFunction)

▷ AddLeftCoclosedMonoidalPostCoComposeMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoclosedMonoidalPostCoComposeMorphism.  $F : (a, b, c) \mapsto \text{LeftCoclosedMonoidalPostCoComposeMorphism}(a, b, c)$ .

### 1.14.160 AddLeftCoclosedMonoidalPostCoComposeMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddLeftCoclosedMonoidalPostCoComposeMorphismWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoclosedMonoidalPostCoComposeMorphismWithGivenObjects.  $F : (s, a, b, c, r) \mapsto \text{LeftCoclosedMonoidalPostCoComposeMorphismWithGivenObjects}(s, a, b, c, r)$ .

### 1.14.161 AddLeftCoclosedMonoidalPreCoComposeMorphism (for IsCapCategory, IsFunction)

▷ AddLeftCoclosedMonoidalPreCoComposeMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoclosedMonoidalPreCoComposeMorphism.  $F : (a, b, c) \mapsto \text{LeftCoclosedMonoidalPreCoComposeMorphism}(a, b, c)$ .

### 1.14.162 AddLeftCoclosedMonoidalPreCoComposeMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddLeftCoclosedMonoidalPreCoComposeMorphismWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftCoclosedMonoidalPreCoComposeMorphismWithGivenObjects.  $F : (s, a, b, c, r) \mapsto \text{LeftCoclosedMonoidalPreCoComposeMorphismWithGivenObjects}(s, a, b, c, r)$ .

### 1.14.163 AddLeftInternalCoHomOnMorphisms (for IsCapCategory, IsFunction)

▷ AddLeftInternalCoHomOnMorphisms( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftInternalCoHomOnMorphisms.  $F : (\alpha, \beta) \mapsto \text{LeftInternalCoHomOnMorphisms}(\alpha, \beta)$ .

### 1.14.164 AddLeftInternalCoHomOnMorphismsWithGivenLeftInternalCoHoms (for IsCapCategory, IsFunction)

▷ AddLeftInternalCoHomOnMorphismsWithGivenLeftInternalCoHoms( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftInternalCoHomOnMorphismsWithGivenLeftInternalCoHoms.  $F : (s, \alpha, \beta, r) \mapsto \text{LeftInternalCoHomOnMorphismsWithGivenLeftInternalCoHoms}(s, \alpha, \beta, r)$ .

### 1.14.165 AddLeftInternalCoHomOnObjects (for IsCapCategory, IsFunction)

▷ AddLeftInternalCoHomOnObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftInternalCoHomOnObjects.  $F : (a, b) \mapsto \text{LeftInternalCoHomOnObjects}(a, b)$ .

### 1.14.166 AddLeftInternalCoHomTensorProductCompatibilityMorphism (for IsCapCategory, IsFunction)

▷ AddLeftInternalCoHomTensorProductCompatibilityMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftInternalCoHomTensorProductCompatibilityMorphism.  $F : (list) \mapsto \text{LeftInternalCoHomTensorProductCompatibilityMorphism}(list)$ .

### 1.14.167 AddLeftInternalCoHomTensorProductCompatibilityMorphismWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddLeftInternalCoHomTensorProductCompatibilityMorphismWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftInternalCoHomTensorProductCompatibilityMorphismWithGivenObjects.  $F : (source, list, range) \mapsto \text{LeftInternalCoHomTensorProductCompatibilityMorphismWithGivenObjects}(source, list, range)$ .

### 1.14.168 AddLeftInternalCoHomToTensorProductAdjunctMorphism (for IsCapCategory, IsFunction)

▷ AddLeftInternalCoHomToTensorProductAdjunctMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftInternalCoHomToTensorProductAdjunctMorphism.  $F : (a, c, f) \mapsto \text{LeftInternalCoHomToTensorProductAdjunctMorphism}(a, c, f)$ .

### 1.14.169 AddLeftInternalCoHomToTensorProductAdjunctMorphismWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ AddLeftInternalCoHomToTensorProductAdjunctMorphismWithGivenTensorProduct( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftInternalCoHomToTensorProductAdjunctMorphismWithGivenTensorProduct.  $F : (a, c, f, t) \mapsto \text{LeftInternalCoHomToTensorProductAdjunctMorphismWithGivenTensorProduct}(a, c, f, t)$ .

### 1.14.170 AddMorphismFromLeftCoBidual (for IsCapCategory, IsFunction)

▷ AddMorphismFromLeftCoBidual( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromLeftCoBidual`.  $F : (a) \mapsto \text{MorphismFromLeftCoBidual}(a)$ .

#### 1.14.171 **AddMorphismFromLeftCoBidualWithGivenLeftCoBidual** (for **IsCapCategory, IsFunction**)

▷ `AddMorphismFromLeftCoBidualWithGivenLeftCoBidual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromLeftCoBidualWithGivenLeftCoBidual`.  $F : (a,s) \mapsto \text{MorphismFromLeftCoBidualWithGivenLeftCoBidual}(a,s)$ .

#### 1.14.172 **AddMorphismFromLeftInternalCoHomToTensorProduct** (for **IsCapCategory, IsFunction**)

▷ `AddMorphismFromLeftInternalCoHomToTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromLeftInternalCoHomToTensorProduct`.  $F : (a,b) \mapsto \text{MorphismFromLeftInternalCoHomToTensorProduct}(a,b)$ .

#### 1.14.173 **AddMorphismFromLeftInternalCoHomToTensorProductWithGivenObjects** (for **IsCapCategory, IsFunction**)

▷ `AddMorphismFromLeftInternalCoHomToTensorProductWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromLeftInternalCoHomToTensorProductWithGivenObjects`.  $F : (s,a,b,r) \mapsto \text{MorphismFromLeftInternalCoHomToTensorProductWithGivenObjects}(s,a,b,r)$ .

#### 1.14.174 **AddTensorProductToLeftInternalCoHomAdjunctMorphism** (for **IsCapCategory, IsFunction**)

▷ `AddTensorProductToLeftInternalCoHomAdjunctMorphism(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductToLeftInternalCoHomAdjunctMorphism`.  $F : (b,c,g) \mapsto \text{TensorProductToLeftInternalCoHomAdjunctMorphism}(b,c,g)$ .

#### 1.14.175 **AddTensorProductToLeftInternalCoHomAdjunctMorphismWithGivenLeftInternalCoHom** (for **IsCapCategory, IsFunction**)

▷ `AddTensorProductToLeftInternalCoHomAdjunctMorphismWithGivenLeftInternalCoHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `TensorProductToLeftInternalCoHomAdjunctMorphismWithGivenLeftInternalCoHom`.  $F : (b, c, g, i) \mapsto \text{TensorProductToLeftInternalCoHomAdjunctMorphismWithGivenLeftInternalCoHom}(b, c, g, i)$ .

#### 1.14.176 AddUniversalPropertyOfLeftCoDual (for IsCapCategory, IsFunction)

▷ `AddUniversalPropertyOfLeftCoDual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `UniversalPropertyOfLeftCoDual`.  $F : (t, a, \alpha) \mapsto \text{UniversalPropertyOfLeftCoDual}(t, a, \alpha)$ .

#### 1.14.177 AddAssociatorLeftToRight (for IsCapCategory, IsFunction)

▷ `AddAssociatorLeftToRight(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `AssociatorLeftToRight`.  $F : (a, b, c) \mapsto \text{AssociatorLeftToRight}(a, b, c)$ .

#### 1.14.178 AddAssociatorLeftToRightWithGivenTensorProducts (for IsCapCategory, IsFunction)

▷ `AddAssociatorLeftToRightWithGivenTensorProducts(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `AssociatorLeftToRightWithGivenTensorProducts`.  $F : (s, a, b, c, r) \mapsto \text{AssociatorLeftToRightWithGivenTensorProducts}(s, a, b, c, r)$ .

#### 1.14.179 AddAssociatorRightToLeft (for IsCapCategory, IsFunction)

▷ `AddAssociatorRightToLeft(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `AssociatorRightToLeft`.  $F : (a, b, c) \mapsto \text{AssociatorRightToLeft}(a, b, c)$ .

#### 1.14.180 AddAssociatorRightToLeftWithGivenTensorProducts (for IsCapCategory, IsFunction)

▷ `AddAssociatorRightToLeftWithGivenTensorProducts(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `AssociatorRightToLeftWithGivenTensorProducts`.  $F : (s, a, b, c, r) \mapsto \text{AssociatorRightToLeftWithGivenTensorProducts}(s, a, b, c, r)$ .



**1.14.181 AddLeftUnitor (for IsCapCategory, IsFunction)**

▷ AddLeftUnitor( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftUnitor.  $F : (a) \mapsto \text{LeftUnitor}(a)$ .

**1.14.182 AddLeftUnitorInverse (for IsCapCategory, IsFunction)**

▷ AddLeftUnitorInverse( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftUnitorInverse.  $F : (a) \mapsto \text{LeftUnitorInverse}(a)$ .

**1.14.183 AddLeftUnitorInverseWithGivenTensorProduct (for IsCapCategory, IsFunction)**

▷ AddLeftUnitorInverseWithGivenTensorProduct( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftUnitorInverseWithGivenTensorProduct.  $F : (a, r) \mapsto \text{LeftUnitorInverseWithGivenTensorProduct}(a, r)$ .

**1.14.184 AddLeftUnitorWithGivenTensorProduct (for IsCapCategory, IsFunction)**

▷ AddLeftUnitorWithGivenTensorProduct( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation LeftUnitorWithGivenTensorProduct.  $F : (a, s) \mapsto \text{LeftUnitorWithGivenTensorProduct}(a, s)$ .

**1.14.185 AddRightUnitor (for IsCapCategory, IsFunction)**

▷ AddRightUnitor( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation RightUnitor.  $F : (a) \mapsto \text{RightUnitor}(a)$ .

**1.14.186 AddRightUnitorInverse (for IsCapCategory, IsFunction)**

▷ AddRightUnitorInverse( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation RightUnitorInverse.  $F : (a) \mapsto \text{RightUnitorInverse}(a)$ .

### 1.14.187 AddRightUnitorInverseWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ AddRightUnitorInverseWithGivenTensorProduct( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation RightUnitorInverseWithGivenTensorProduct.  $F : (a, r) \mapsto \text{RightUnitorInverseWithGivenTensorProduct}(a, r)$ .

### 1.14.188 AddRightUnitorWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ AddRightUnitorWithGivenTensorProduct( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation RightUnitorWithGivenTensorProduct.  $F : (a, s) \mapsto \text{RightUnitorWithGivenTensorProduct}(a, s)$ .

### 1.14.189 AddTensorProductOnMorphisms (for IsCapCategory, IsFunction)

▷ AddTensorProductOnMorphisms( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductOnMorphisms.  $F : (\alpha, \beta) \mapsto \text{TensorProductOnMorphisms}(\alpha, \beta)$ .

### 1.14.190 AddTensorProductOnMorphismsWithGivenTensorProducts (for IsCapCategory, IsFunction)

▷ AddTensorProductOnMorphismsWithGivenTensorProducts( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductOnMorphismsWithGivenTensorProducts.  $F : (s, \alpha, \beta, r) \mapsto \text{TensorProductOnMorphismsWithGivenTensorProducts}(s, \alpha, \beta, r)$ .

### 1.14.191 AddCoevaluationForDual (for IsCapCategory, IsFunction)

▷ AddCoevaluationForDual( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoevaluationForDual.  $F : (a) \mapsto \text{CoevaluationForDual}(a)$ .

### 1.14.192 AddCoevaluationForDualWithGivenTensorProduct (for IsCapCategory, IsFunction)

▷ AddCoevaluationForDualWithGivenTensorProduct( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `CoevaluationForDualWithGivenTensorProduct`.  $F : (s, a, r) \mapsto \text{CoevaluationForDualWithGivenTensorProduct}(s, a, r)$ .

#### 1.14.193 AddIsomorphismFromInternalHomToTensorProductWithDualObject (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromInternalHomToTensorProductWithDualObject(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromInternalHomToTensorProductWithDualObject`.  $F : (a, b) \mapsto \text{IsomorphismFromInternalHomToTensorProductWithDualObject}(a, b)$ .

#### 1.14.194 AddIsomorphismFromTensorProductWithDualObjectToInternalHom (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromTensorProductWithDualObjectToInternalHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromTensorProductWithDualObjectToInternalHom`.  $F : (a, b) \mapsto \text{IsomorphismFromTensorProductWithDualObjectToInternalHom}(a, b)$ .

#### 1.14.195 AddMorphismFromBidual (for IsCapCategory, IsFunction)

▷ `AddMorphismFromBidual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromBidual`.  $F : (a) \mapsto \text{MorphismFromBidual}(a)$ .

#### 1.14.196 AddMorphismFromBidualWithGivenBidual (for IsCapCategory, IsFunction)

▷ `AddMorphismFromBidualWithGivenBidual(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromBidualWithGivenBidual`.  $F : (a, s) \mapsto \text{MorphismFromBidualWithGivenBidual}(a, s)$ .

#### 1.14.197 AddMorphismFromInternalHomToTensorProduct (for IsCapCategory, IsFunction)

▷ `AddMorphismFromInternalHomToTensorProduct(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromInternalHomToTensorProduct`.  $F : (a, b) \mapsto \text{MorphismFromInternalHomToTensorProduct}(a, b)$ .

### 1.14.198 AddMorphismFromInternalHomToTensorProductWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddMorphismFromInternalHomToTensorProductWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation MorphismFromInternalHomToTensorProductWithGivenObjects.  $F : (s, a, b, r) \mapsto \text{MorphismFromInternalHomToTensorProductWithGivenObjects}(s, a, b, r)$ .

### 1.14.199 AddRankMorphism (for IsCapCategory, IsFunction)

▷ AddRankMorphism( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation RankMorphism.  $F : (a) \mapsto \text{RankMorphism}(a)$ .

### 1.14.200 AddTensorProductInternalHomCompatibilityMorphismInverse (for IsCapCategory, IsFunction)

▷ AddTensorProductInternalHomCompatibilityMorphismInverse( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductInternalHomCompatibilityMorphismInverse.  $F : (list) \mapsto \text{TensorProductInternalHomCompatibilityMorphismInverse}(list)$ .

### 1.14.201 AddTensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects (for IsCapCategory, IsFunction)

▷ AddTensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects.  $F : (source, list, range) \mapsto \text{TensorProductInternalHomCompatibilityMorphismInverseWithGivenObjects}(source, list, range)$ .

### 1.14.202 AddTraceMap (for IsCapCategory, IsFunction)

▷ AddTraceMap( $C, F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation TraceMap.  $F : (alpha) \mapsto \text{TraceMap}(alpha)$ .

**1.14.203 AddCoRankMorphism (for IsCapCategory, IsFunction)**

▷ AddCoRankMorphism( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoRankMorphism.  $F : (a) \mapsto \text{CoRankMorphism}(a)$ .

**1.14.204 AddCoTraceMap (for IsCapCategory, IsFunction)**

▷ AddCoTraceMap( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoTraceMap.  $F : (\alpha) \mapsto \text{CoTraceMap}(\alpha)$ .

**1.14.205 AddCoclosedCoevaluationForCoDual (for IsCapCategory, IsFunction)**

▷ AddCoclosedCoevaluationForCoDual( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoclosedCoevaluationForCoDual.  $F : (a) \mapsto \text{CoclosedCoevaluationForCoDual}(a)$ .

**1.14.206 AddCoclosedCoevaluationForCoDualWithGivenTensorProduct (for IsCapCategory, IsFunction)**

▷ AddCoclosedCoevaluationForCoDualWithGivenTensorProduct( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation CoclosedCoevaluationForCoDualWithGivenTensorProduct.  $F : (s, a, r) \mapsto \text{CoclosedCoevaluationForCoDualWithGivenTensorProduct}(s, a, r)$ .

**1.14.207 AddInternalCoHomTensorProductCompatibilityMorphismInverse (for IsCapCategory, IsFunction)**

▷ AddInternalCoHomTensorProductCompatibilityMorphismInverse( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation InternalCoHomTensorProductCompatibilityMorphismInverse.  $F : (list) \mapsto \text{InternalCoHomTensorProductCompatibilityMorphismInverse}(list)$ .

**1.14.208 AddInternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects (for IsCapCategory, IsFunction)**

▷ AddInternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects( $C$ ,  $F$ ) (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `InternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects`.  $F : (source, list, range) \mapsto \text{InternalCoHomTensorProductCompatibilityMorphismInverseWithGivenObjects}(source, list, range)$

#### 1.14.209 AddIsomorphismFromInternalCoHomToTensorProductWithCoDualObject (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromInternalCoHomToTensorProductWithCoDualObject(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromInternalCoHomToTensorProductWithCoDualObject`.  $F : (a, b) \mapsto \text{IsomorphismFromInternalCoHomToTensorProductWithCoDualObject}(a, b)$ .

#### 1.14.210 AddIsomorphismFromTensorProductWithCoDualObjectToInternalCoHom (for IsCapCategory, IsFunction)

▷ `AddIsomorphismFromTensorProductWithCoDualObjectToInternalCoHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `IsomorphismFromTensorProductWithCoDualObjectToInternalCoHom`.  $F : (a, b) \mapsto \text{IsomorphismFromTensorProductWithCoDualObjectToInternalCoHom}(a, b)$ .

#### 1.14.211 AddMorphismFromTensorProductToInternalCoHom (for IsCapCategory, IsFunction)

▷ `AddMorphismFromTensorProductToInternalCoHom(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromTensorProductToInternalCoHom`.  $F : (a, b) \mapsto \text{MorphismFromTensorProductToInternalCoHom}(a, b)$ .

#### 1.14.212 AddMorphismFromTensorProductToInternalCoHomWithGivenObjects (for IsCapCategory, IsFunction)

▷ `AddMorphismFromTensorProductToInternalCoHomWithGivenObjects(C, F)` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismFromTensorProductToInternalCoHomWithGivenObjects`.  $F : (s, a, b, r) \mapsto \text{MorphismFromTensorProductToInternalCoHomWithGivenObjects}(s, a, b, r)$ .

**1.14.213 AddMorphismToCoBidual (for IsCapCategory, IsFunction)**

▷ `AddMorphismToCoBidual( $C$ ,  $F$ )` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismToCoBidual`.  $F : (a) \mapsto \text{MorphismToCoBidual}(a)$ .

**1.14.214 AddMorphismToCoBidualWithGivenCoBidual (for IsCapCategory, IsFunction)**

▷ `AddMorphismToCoBidualWithGivenCoBidual( $C$ ,  $F$ )` (operation)

**Returns:** nothing

The arguments are a category  $C$  and a function  $F$ . This operation adds the given function  $F$  to the category for the basic operation `MorphismToCoBidualWithGivenCoBidual`.  $F : (a, r) \mapsto \text{MorphismToCoBidualWithGivenCoBidual}(a, r)$ .

## Chapter 2

# Examples and Tests

### 2.1 Test functions

#### 2.1.1 AdditiveMonoidalCategoriesTest

▷ `AdditiveMonoidalCategoriesTest(cat, a, L)` (function)

The arguments are

- a CAP category *cat*
- an object *a*
- a list *L* of objects

This function checks for every operation declared in `AdditiveMonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

#### 2.1.2 BraidedMonoidalCategoriesTest

▷ `BraidedMonoidalCategoriesTest(cat, a, b)` (function)

The arguments are

- a CAP category *cat*
- objects *a, b*

This function checks for every operation declared in `BraidedMonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options



- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.3 ClosedMonoidalCategoriesTest

▷ `ClosedMonoidalCategoriesTest(cat, a, b, c, d, alpha, beta, gamma, delta, epsilon, zeta)` (function)

The arguments are

- a CAP category *cat*
- objects  $a, b, c, d$
- a morphism  $\alpha : a \rightarrow b$
- a morphism  $\beta : c \rightarrow d$
- a morphism  $\gamma : a \otimes b \rightarrow 1$
- a morphism  $\delta : c \otimes d \rightarrow 1$
- a morphism  $\varepsilon : 1 \rightarrow \text{Hom}(a, b)$
- a morphism  $\zeta : 1 \rightarrow \text{Hom}(c, d)$

This function checks for every operation declared in `ClosedMonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.4 ClosedMonoidalCategoriesTestWithGiven

▷ `ClosedMonoidalCategoriesTestWithGiven(cat, a, b, c, d, alpha, beta)` (function)

The arguments are

- a CAP category *cat*
- objects  $a, b, c, d$
- a morphism  $\alpha : a \rightarrow b$

- a morphism  $\beta : c \rightarrow d$

This function checks for some `*WithGiven` operations declared in `ClosedMonoidalCategories.gd` if they are computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.5 CoclosedMonoidalCategoriesTest

▷ `CoclosedMonoidalCategoriesTest(cat, a, b, c, d, alpha, beta, gamma, delta, epsilon, zeta)` (function)

The arguments are

- a CAP category *cat*
- objects  $a, b, c, d$
- a morphism  $\alpha : a \rightarrow b$
- a morphism  $\beta : c \rightarrow d$
- a morphism  $\gamma : 1 \rightarrow a \otimes b$
- a morphism  $\delta : 1 \rightarrow c \otimes d$
- a morphism  $\varepsilon : \text{coHom}(a, b) \rightarrow 1$
- a morphism  $\zeta : \text{coHom}(c, d) \rightarrow 1$

This function checks for every operation declared in `CoclosedMonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.6 CoclosedMonoidalCategoriesTestWithGiven

▷ `CoclosedMonoidalCategoriesTestWithGiven(cat, a, b, c, d, alpha, beta, gamma, delta, epsilon, zeta)` (function)

The arguments are

- a CAP category *cat*
- objects  $a, b, c, d$
- a morphism  $\alpha : a \rightarrow b$
- a morphism  $\beta : c \rightarrow d$

This function checks for some `*WithGiven` operations declared in `CoclosedMonoidalCategories.gd` if they are computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.7 LeftClosedMonoidalCategoriesTest

▷ `LeftClosedMonoidalCategoriesTest(cat, a, b, c, d, alpha, beta, gamma, delta, epsilon, zeta)` (function)

The arguments are

- a CAP category *cat*
- objects  $a, b, c, d$
- a morphism  $\alpha : a \rightarrow b$
- a morphism  $\beta : c \rightarrow d$
- a morphism  $\gamma : a \otimes b \rightarrow 1$
- a morphism  $\delta : c \otimes d \rightarrow 1$
- a morphism  $\varepsilon : 1 \rightarrow \text{Hom}(a, b)$
- a morphism  $\zeta : 1 \rightarrow \text{Hom}(c, d)$

This function checks for every operation declared in `LeftClosedMonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in `cat`. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.8 LeftClosedMonoidalCategoriesTestWithGiven

▷ `LeftClosedMonoidalCategoriesTestWithGiven(cat, a, b, c, d, alpha, beta, gamma, delta, epsilon, zeta)` (function)

The arguments are

- a CAP category `cat`
- objects  $a, b, c, d$
- a morphism  $\alpha : a \rightarrow b$
- a morphism  $\beta : c \rightarrow d$

This function checks for some `*WithGiven` operationS declared in `LeftClosedMonoidalCategories.gd` if they are computable in the CAP category `cat`. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of `cat`. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in `cat`. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.9 LeftCoclosedMonoidalCategoriesTest

▷ `LeftCoclosedMonoidalCategoriesTest(cat, a, b, c, d, alpha, beta, gamma, delta, epsilon, zeta)` (function)

The arguments are

- a CAP category `cat`
- objects  $a, b, c, d$
- a morphism  $\alpha : a \rightarrow b$
- a morphism  $\beta : c \rightarrow d$
- a morphism  $\gamma : 1 \rightarrow a \otimes b$
- a morphism  $\delta : 1 \rightarrow c \otimes d$

- a morphism  $\varepsilon : \text{coHom}(a, b) \rightarrow 1$
- a morphism  $\zeta : \text{coHom}(c, d) \rightarrow 1$

This function checks for every operation declared in `LeftCoclosedMonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.10 LeftCoclosedMonoidalCategoriesTestWithGiven

▷ `LeftCoclosedMonoidalCategoriesTestWithGiven(cat, a, b, c, d, alpha, beta, gamma, delta, epsilon, zeta)` (function)

The arguments are

- a CAP category *cat*
- objects *a, b, c, d*
- a morphism  $\alpha : a \rightarrow b$
- a morphism  $\beta : c \rightarrow d$

This function checks for some `*WithGiven` operations declared in `LeftCoclosedMonoidalCategories.gd` if they are computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.11 MonoidalCategoriesTensorProductAndUnitTest

▷ `MonoidalCategoriesTensorProductAndUnitTest(cat, a, b)` (function)

The arguments are

- a CAP category *cat*
- objects *a, b*

This function checks for every operation declared in `MonoidalCategoriesTensorProductAndUnit.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.12 MonoidalCategoriesTest

▷ `MonoidalCategoriesTest(cat, a, b, c, alpha, beta)` (function)

The arguments are

- a CAP category *cat*
- objects *a, b, c*
- a morphism  $\alpha : a \rightarrow b$
- a morphism  $\beta : c \rightarrow d$

This function checks for every operation declared in `MonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in *cat*. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.13 RigidSymmetricClosedMonoidalCategoriesTest

▷ `RigidSymmetricClosedMonoidalCategoriesTest(cat, a, b, c, d, alpha)` (function)

The arguments are

- a CAP category *cat*
- objects *a, b, c, d*
- an endomorphism  $\alpha : a \rightarrow a$

This function checks for every object and morphism declared in `RigidSymmetricClosedMonoidalCategories.gd` if it is computable in the CAP category *cat*. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of *cat*. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in `cat`. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

### 2.1.14 RigidSymmetricCoclosedMonoidalCategoriesTest

▷ `RigidSymmetricCoclosedMonoidalCategoriesTest(cat, a, b, c, d, alpha)` (function)

The arguments are

- a CAP category `cat`
- objects `a, b, c, d`
- an endomorphism  $\alpha : a \rightarrow a$

This function checks for every object and morphism declared in `RigidSymmetricCoclosedMonoidalCategories.gd` if it is computable in the CAP category `cat`. If yes, then the operation is executed with the parameters given above and compared to the equivalent computation in the opposite category of `cat`. Pass the options

- `verbose := true` to output more information.
- `only_primitive_operations := true`, which is passed on to `Opposite()`, to only primitively install dual operations for primitively installed operations in `cat`. The advantage is, that more derivations might be tested. On the downside, this might test fewer `dual_pre/postprocessor_funcs`.

## 2.2 Basics

Example

```
gap> LoadPackage( "MonoidalCategories", false );
true
gap> LoadPackage( "LinearAlgebraForCAP", false );
true
gap> Q := HomalgFieldOfRationals();
Q
gap> a := VectorSpaceObject( 1, Q );
<A vector space object over Q of dimension 1>
gap> b := VectorSpaceObject( 2, Q );
<A vector space object over Q of dimension 2>
gap> c := VectorSpaceObject( 3, Q );
<A vector space object over Q of dimension 3>
gap> z := ZeroObject( CapCategory( a ) );
<A vector space object over Q of dimension 0>
gap> alpha := VectorSpaceMorphism( a, [ [ 1, 0 ] ], b );
<A morphism in Category of matrices over Q>
gap> beta := VectorSpaceMorphism( b,
> [ [ 1, 0, 0 ], [ 0, 1, 0 ] ], c );
```

```

<A morphism in Category of matrices over Q>
gap> gamma := VectorSpaceMorphism( c,
>      [ [ 0, 1, 1 ], [ 1, 0, 1 ], [ 1, 1, 0 ] ], c );
<A morphism in Category of matrices over Q>
gap> IsCongruentForMorphisms(
>      TensorProductOnMorphisms( alpha, beta ),
>      TensorProductOnMorphisms( beta, alpha ) );
false
gap> IsOne( AssociatorRightToLeft( a, b, c ) );
true
gap> IsCongruentForMorphisms(
>      gamma, LambdaElimination( c, c, LambdaIntroduction( gamma ) ) );
true
gap> IsZero( TraceMap( gamma ) );
true
gap> IsCongruentForMorphisms(
>      RankMorphism( DirectSum( a, b ) ), RankMorphism( c ) );
true
gap> IsOne( Braiding( b, c ) );
false
gap> IsOne( PreCompose( Braiding( b, c ), Braiding( c, b ) ) );
true

```



## Chapter 3

# Code Generation for Monoidal Categories

### 3.1 Monoidal Categories

#### 3.1.1 WriteFileForMonoidalStructure

▷ WriteFileForMonoidalStructure(*key\_val\_rec*, *package\_name*, *files\_rec*) (function)

**Returns:** nothing

This functions uses the dictionary *key\_val\_rec* to create a new monoidal structure. It generates the necessary files in the package *package\_name* using the file-correspondence table *files\_rec*. See the implementation for details.

### 3.2 Closed Monoidal Categories

#### 3.2.1 WriteFileForClosedMonoidalStructure

▷ WriteFileForClosedMonoidalStructure(*key\_val\_rec*, *package\_name*, *files\_rec*) (function)

**Returns:** nothing

This functions uses the dictionary *key\_val\_rec* to create a new closed monoidal structure. It generates the necessary files in the package *package\_name* using the file-correspondence table *files\_rec*. See the implementation for details.

#### 3.2.2 WriteFileForLeftClosedMonoidalStructure

▷ WriteFileForLeftClosedMonoidalStructure(*key\_val\_rec*, *package\_name*, *files\_rec*) (function)

**Returns:** nothing

This functions uses the dictionary *key\_val\_rec* to create a new left closed monoidal structure. It generates the necessary files in the package *package\_name* using the file-correspondence table *files\_rec*. See the implementation for details.

### 3.3 Coclosed Monoidal Categories

#### 3.3.1 WriteFileForCoclosedMonoidalStructure

▷ `WriteFileForCoclosedMonoidalStructure(key_val_rec, package_name, files_rec)`  
(function)

**Returns:** nothing

This functions uses the dictionary `key_val_rec` to create a new coclosed monoidal structure. It generates the necessary files in the package `package_name` using the file-correspondence table `files_rec`. See the implementation for details.

#### 3.3.2 WriteFileForLeftCoclosedMonoidalStructure

▷ `WriteFileForLeftCoclosedMonoidalStructure(key_val_rec, package_name, files_rec)`  
(function)

**Returns:** nothing

This functions uses the dictionary `key_val_rec` to create a new left coclosed monoidal structure. It generates the necessary files in the package `package_name` using the file-correspondence table `files_rec`. See the implementation for details.

## Chapter 4

# The terminal category with multiple objects

This is an example of a category which is created using `CategoryConstructor` out of no input.

This category “lies” in all doctrines and can hence be used (in conjunction with `LazyCategory`) in order to check the type-correctness of the various derived methods provided by `CAP` or any `CAP`-based package.

### 4.1 Constructors

### 4.2 `GAP` Categories

## Chapter 5

# Legacy Operations and Synonyms

### 5.1 Legacy operations

#### 5.1.1 `CoclosedCoevaluationMorphism` (for `IsCapCategoryObject`, `IsCapCategoryObject`)

▷ `CoclosedCoevaluationMorphism(a, b)` (operation)

This is a legacy operation for `CoclosedMonoidalLeftCoevaluationMorphism(b, a)`, i.e., with the first and second argument interchanged.

#### 5.1.2 `CoclosedCoevaluationMorphismWithGivenSource` (for `IsCapCategoryObject`, `IsCapCategoryObject`, `IsCapCategoryObject`)

▷ `CoclosedCoevaluationMorphismWithGivenSource(a, b, s)` (operation)

This is a legacy operation for `CoclosedMonoidalLeftCoevaluationMorphismWithGivenSource(b, a, s)`, i.e., with the first and second argument interchanged.

#### 5.1.3 `CoclosedEvaluationMorphism` (for `IsCapCategoryObject`, `IsCapCategoryObject`)

▷ `CoclosedEvaluationMorphism(a, b)` (operation)

This is a legacy operation for `CoclosedMonoidalLeftEvaluationMorphism(b, a)`, i.e., with the first and second argument interchanged.

#### 5.1.4 `CoclosedEvaluationMorphismWithGivenRange` (for `IsCapCategoryObject`, `IsCapCategoryObject`, `IsCapCategoryObject`)

▷ `CoclosedEvaluationMorphismWithGivenRange(a, b, r)` (operation)

This is a legacy operation for `CoclosedMonoidalLeftEvaluationMorphismWithGivenRange(b, a, r)`, i.e., with the first and second argument interchanged.

### 5.1.5 CoevaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ CoevaluationMorphism(a, b) (operation)

This is a legacy operation for ClosedMonoidalLeftCoevaluationMorphism( b, a ), i.e., with the first and second argument interchanged.

### 5.1.6 CoevaluationMorphismWithGivenRange (for IsCapCategoryObject, IsCapCategoryObject)

▷ CoevaluationMorphismWithGivenRange(a, b, r) (operation)

This is a legacy operation for ClosedMonoidalLeftCoevaluationMorphismWithGivenRange( b, a, r ), i.e., with the first and second argument interchanged.

## 5.2 Synonyms for legacy operations

### 5.2.1 EvaluationMorphism (for IsCapCategoryObject, IsCapCategoryObject)

▷ EvaluationMorphism(arg1, arg2) (operation)

This is a synonym for ClosedMonoidalLeftEvaluationMorphism.

### 5.2.2 EvaluationMorphismWithGivenSource (for IsCapCategoryObject, IsCapCategoryObject, IsCapCategoryObject)

▷ EvaluationMorphismWithGivenSource(arg1, arg2, arg3) (operation)

This is a synonym for ClosedMonoidalLeftEvaluationMorphismWithGivenSource.

### 5.2.3 InternalCoHomToTensorProductAdjunctionMap (for IsObject)

▷ InternalCoHomToTensorProductAdjunctionMap(arg) (operation)

This is a synonym for InternalCoHomToTensorProductLeftAdjunctMorphism.

### 5.2.4 InternalCoHomToTensorProductAdjunctionMapWithGivenTensorProduct (for IsObject)

▷ InternalCoHomToTensorProductAdjunctionMapWithGivenTensorProduct(arg) (operation)

This is a synonym for InternalCoHomToTensorProductLeftAdjunctionMapWithGivenTensorProduct.

### 5.2.5 InternalHomToTensorProductAdjunctionMap (for IsObject)

▷ InternalHomToTensorProductAdjunctionMap(*arg*) (operation)

This is a synonym for InternalHomToTensorProductLeftAdjunctMorphism.

### 5.2.6 InternalHomToTensorProductAdjunctionMapWithGivenTensorProduct (for IsObject)

▷ InternalHomToTensorProductAdjunctionMapWithGivenTensorProduct(*arg*) (operation)

This is a synonym for InternalHomToTensorProductLeftAdjunctionMapWithGivenTensorProduct.

### 5.2.7 TensorProductToInternalCoHomAdjunctionMap (for IsObject)

▷ TensorProductToInternalCoHomAdjunctionMap(*arg*) (operation)

This is a synonym for TensorProductToInternalCoHomLeftAdjunctMorphism.

### 5.2.8 TensorProductToInternalCoHomAdjunctionMapWithGivenInternalCoHom (for IsObject)

▷ TensorProductToInternalCoHomAdjunctionMapWithGivenInternalCoHom(*arg*) (operation)

This is a synonym for TensorProductToInternalCoHomLeftAdjunctMorphismWithGivenInternalCoHom.

### 5.2.9 TensorProductToInternalHomAdjunctionMap (for IsObject)

▷ TensorProductToInternalHomAdjunctionMap(*arg*) (operation)

This is a synonym for TensorProductToInternalHomLeftAdjunctMorphism.

### 5.2.10 TensorProductToInternalHomAdjunctionMapWithGivenInternalHom (for IsObject)

▷ TensorProductToInternalHomAdjunctionMapWithGivenInternalHom(*arg*) (operation)

This is a synonym for TensorProductToInternalHomLeftAdjunctMorphismWithGivenInternalHom.

### 5.2.11 InternalCoHomToTensorProductLeftAdjunctionMap (for IsObject)

▷ InternalCoHomToTensorProductLeftAdjunctionMap(*arg*) (operation)

This is a synonym for InternalCoHomToTensorProductLeftAdjunctMorphism.

### 5.2.12 InternalHomToTensorProductLeftAdjunctionMap (for IsObject)

▷ `InternalHomToTensorProductLeftAdjunctionMap(arg)` (operation)

This is a synonym for `InternalHomToTensorProductLeftAdjunctMorphism`.

### 5.2.13 TensorProductToInternalCoHomLeftAdjunctionMap (for IsObject)

▷ `TensorProductToInternalCoHomLeftAdjunctionMap(arg)` (operation)

This is a synonym for `TensorProductToInternalCoHomLeftAdjunctMorphism`.

### 5.2.14 TensorProductToInternalCoHomLeftAdjunctionMapWithGivenInternalCoHom (for IsObject)

▷ `TensorProductToInternalCoHomLeftAdjunctionMapWithGivenInternalCoHom(arg)` (operation)

This is a synonym for `TensorProductToInternalCoHomLeftAdjunctMorphismWithGivenInternalCoHom`.

### 5.2.15 TensorProductToInternalHomLeftAdjunctionMap (for IsObject)

▷ `TensorProductToInternalHomLeftAdjunctionMap(arg)` (operation)

This is a synonym for `TensorProductToInternalHomLeftAdjunctMorphism`.

### 5.2.16 TensorProductToInternalHomLeftAdjunctionMapWithGivenInternalHom (for IsObject)

▷ `TensorProductToInternalHomLeftAdjunctionMapWithGivenInternalHom(arg)` (operation)

This is a synonym for `TensorProductToInternalHomLeftAdjunctMorphismWithGivenInternalHom`.

## Chapter 6

# MonoidalCategories automatic generated documentation

### 6.1 MonoidalCategories automatic generated documentation of properties

#### 6.1.1 IsBraidedMonoidalCategory (for IsCapCategory)

▷ `IsBraidedMonoidalCategory( $\mathcal{C}$ )` (property)  
**Returns:** true or false  
The property of the category  $\mathcal{C}$  being braided monoidal.

#### 6.1.2 IsClosedMonoidalCategory (for IsCapCategory)

▷ `IsClosedMonoidalCategory( $\mathcal{C}$ )` (property)  
**Returns:** true or false  
The property of the category  $\mathcal{C}$  being (bi)closed monoidal.

#### 6.1.3 IsCoclosedMonoidalCategory (for IsCapCategory)

▷ `IsCoclosedMonoidalCategory( $\mathcal{C}$ )` (property)  
**Returns:** true or false  
The property of the category  $\mathcal{C}$  being (bi)coclosed monoidal.

#### 6.1.4 IsLeftClosedMonoidalCategory (for IsCapCategory)

▷ `IsLeftClosedMonoidalCategory( $\mathcal{C}$ )` (property)  
**Returns:** true or false  
The property of the category  $\mathcal{C}$  being left closed monoidal.

#### 6.1.5 IsLeftCoclosedMonoidalCategory (for IsCapCategory)

▷ `IsLeftCoclosedMonoidalCategory( $\mathcal{C}$ )` (property)  
**Returns:** true or false  
The property of the category  $\mathcal{C}$  being coclosed monoidal.



### 6.1.6 IsMonoidalCategory (for IsCapCategory)

- ▷ IsMonoidalCategory( $C$ ) (property)  
**Returns:** true or false  
 The property of the category  $C$  being monoidal.

### 6.1.7 IsStrictMonoidalCategory (for IsCapCategory)

- ▷ IsStrictMonoidalCategory( $C$ ) (property)  
**Returns:** true or false  
 The property of the category  $C$  being strict monoidal.

### 6.1.8 IsRigidSymmetricClosedMonoidalCategory (for IsCapCategory)

- ▷ IsRigidSymmetricClosedMonoidalCategory( $C$ ) (property)  
**Returns:** true or false  
 The property of the category  $C$  being rigid symmetric closed monoidal.

### 6.1.9 IsRigidSymmetricCoclosedMonoidalCategory (for IsCapCategory)

- ▷ IsRigidSymmetricCoclosedMonoidalCategory( $C$ ) (property)  
**Returns:** true or false  
 The property of the category  $C$  being rigid symmetric coclosed monoidal.

### 6.1.10 IsSymmetricClosedMonoidalCategory (for IsCapCategory)

- ▷ IsSymmetricClosedMonoidalCategory( $C$ ) (property)  
**Returns:** true or false  
 The property of the category  $C$  being symmetric closed monoidal.

### 6.1.11 IsSymmetricCoclosedMonoidalCategory (for IsCapCategory)

- ▷ IsSymmetricCoclosedMonoidalCategory( $C$ ) (property)  
**Returns:** true or false  
 The property of the category  $C$  being symmetric coclosed monoidal.

### 6.1.12 IsSymmetricMonoidalCategory (for IsCapCategory)

- ▷ IsSymmetricMonoidalCategory( $C$ ) (property)  
**Returns:** true or false  
 The property of the category  $C$  being symmetric monoidal.

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